

Leads in the ice cover of the Arctic Seas and possibilities of forecasting their characteristics on the basis of satellite data

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ABSTRACT

Discontinuities in the ice cover of the Arctic Seas present potential routes of easier navigation under the severe ice conditions. Therefore, to forecast the prevailing orientation of discontinuities and location of zones of their formation and closing is an important objective.

The developed method of forecasting is based on the use of complex analogues. Each of them presents a set of atmospheric pressure fields, ice drift, drift velocity divergence, modal orientation and density of leads.

For the prognostic atmospheric pressure above the sea, a complex analogue with a similar structure of the atmospheric pressure field will be chosen. In this case, the fields of characteristics of leads will be taken from the analogue as prognostic. Based on the divergence values of the ice drift velocity calculated from the analogue, the anticipated zones of compression and divergence in the ice cover will be determined.

In order to have a possibility for a quick choice of an analogue for any atmospheric pressure field, a classification of the types of atmospheric circulation observed in the study area is made for each sea. The classification takes into account the direction of air flows and their intensity.

Based on satellite images, an electronic archive of the actual data on the location of discontinuities in the ice cover of the Arctic Seas is formed from which their characteristics are calculated.

KEY WORDS: Leads; Laptev Sea; Ice forecast.

INTRODUCTION

The discontinuities in the sea ice cover which are also called leads are a characteristic element of the ice cover structure in the Arctic Basin and the Arctic Seas. They are presented as cracks, canals and fractures with open water or young ice. Their length can comprise several hundreds of kilometers and more and their width is 5-10 km. Leads are also formed beyond the Arctic Seas in the deepwater part, which is called the Arctic Basin in scientific literature of the Russian authors.

Practical interest in leads is caused by their possible use in navigation (Brestkin et al., 1995, Mikhailichenko, 1999). According to data (Frolov, 2013), when navigators use leads the orientation of which deviates from the ship general course by not more than 30°, the motion speed increases 1.5-2 fold. Being potential routes of easier navigation, icebreakers use leads for escorting transport ships in the winter-spring period and during solo voyages. There is also a known case of an unescorted passage of a transport vessel to the North Pole (Gorbunov et al., 2008) using systems of leads en-route.

The possibility of advanced planning of optimal navigation routes along the NSR in the winter-spring period taking into account the en-route discontinuities identifies the demand for the forecasting the prevailing orientation of leads and zones of their formation.

At the present time there is a method of forecasting the characteristics of leads in the Kara Sea (Gorbunov et al., 2001). The aim of this study is to develop a method of forecasting the prevailing orientation and zones of formation of leads in the Laptev and East-Siberian Seas.

Leads are a consequence of tensile strain of the ice cover. The issues of deformation were considered in a number of articles. It was proposed to calculate the local indicator of diverging and compression in (Aleksanin et al., 2017) on the basis of satellite data as a rate of change of the distance between separate elements of the ice cover. In (Linow et al., 2015) one describes an approach based on the calculation of strain tensor by derivatives of the field of ice drift velocity, determined from the successive satellite images of the ice cover. There is a method of evaluation of deformation zones by the successive images of RADARSAT-2 satellite (Coon et al., 2007), based on the analysis of leads in the ice cover by the regular grid squares. During an evaluation of the ice cover deformation some authors (Marsan et al, 2004, Weiss and Marsan, 2004) apply a fractal analysis to satellite images.

For an assessment of the possibility of forecasting the characteristics of discontinuities in the ice cover, it is necessary to choose the forecasting method. For this, the hydrodynamic (numerical) and statistical methods of forecasts were considered.

The numerical method is based on the use of equations of hydrodynamics for calculation of changes in time and the data defining the initial conditions and values. The attempts of using the hydrodynamic method of forecasting the characteristics of leads for the Arctic Seas revealed its low efficiency. One of the main causes of low performance of such forecasts is insufficient quantity of initial data for taking into account the factors influencing the processes of formation of leads in the sea ice cover of the Arctic Seas.

When the statistical methods are used, one can predict the future state of ice cover characteristics, having an archived series of observations of the changes of ice and hydrometeorological characteristics. The subject matter of the statistical method is in application of the methods of mathematical statistics and the probability theory to revealing the regularities of ice processes and prognostic connections between different ice and hydrometeorological characteristics. In the framework of this method linear and a non-linear regressions are used, a model by sampling of maximum similarity and other models. This method however does not allow an efficient forecasting of the characteristics of ice cover discontinuities, as the formation of leads is little related to the preceding state of ice and hydrometeorological situation.

METHODOLOGY OF FORECASTING THE CHARACTERISTICS OF LEADS IN THE ICE COVER

The air flows influencing the ice cover form the ice drift fields. The non-uniformity of the ice drift velocity drift crates zones of pressure and diverging in the ice cover, where leads are formed or close (Karelin, 1985, Borodachev, 1988). At intensive compression, ice ridges form in the ice cover, however consideration of ice ridge formation is beyond this article.

Wind depends on the atmospheric pressure gradient, and therefore the pressure fields are a primary predictor for forecasting the lead characteristics. At the changed wind direction, modification of the systems of leads in the ice cover occurs. The direction of leads and their specific length change. The analysis of the surface pressure fields, ice drift and systems of leads shows the dependence of the direction of leads on the atmospheric pressure field and the related ice drift field (Losev and Gorbunov, 1998).

Fig. 1a shows the atmospheric pressure field above the Laptev Sea on 23 December of 2017. As is known, wind deviates from isobars towards low pressure. At the latitude of the Laptev Sea such deviation is insignificant, about 8 degrees. The ice drift from the Arctic Basin is directed southward, as shown in Fig. 1b. Although the ice drift deviates from the wind direction under the impact of the Coriolis force to the right, however in the given case the influence of the shores contributed to the ice drift being close to meridional. Fig. 1c presents leads in the ice cover,

formed by such drift. The prevailing direction of leads is close to the perpendicular drift direction, although no full coincidence is observed. If one considers that, the formation of leads is mainly determined by the drift, then synchronization by time of the drift and formation of leads is important. The field of leads is fixed by satellite images. An ideal case is where all leads within the entire sea were determined at one moment of time and the drift was determined for a time interval of several hours, including the moment of image creation. If the period of time for which the ice drift was determined was long, then the resulting drift vector can differ significantly from vectors for some small time segments in this period. Then the orientation of leads can correspond not to the resulting drift presented in Fig. 1b, but to the drift for a shorter period. Probably this explains the non-coincidence of the drift vectors and orientation of leads in some regions of the sea.

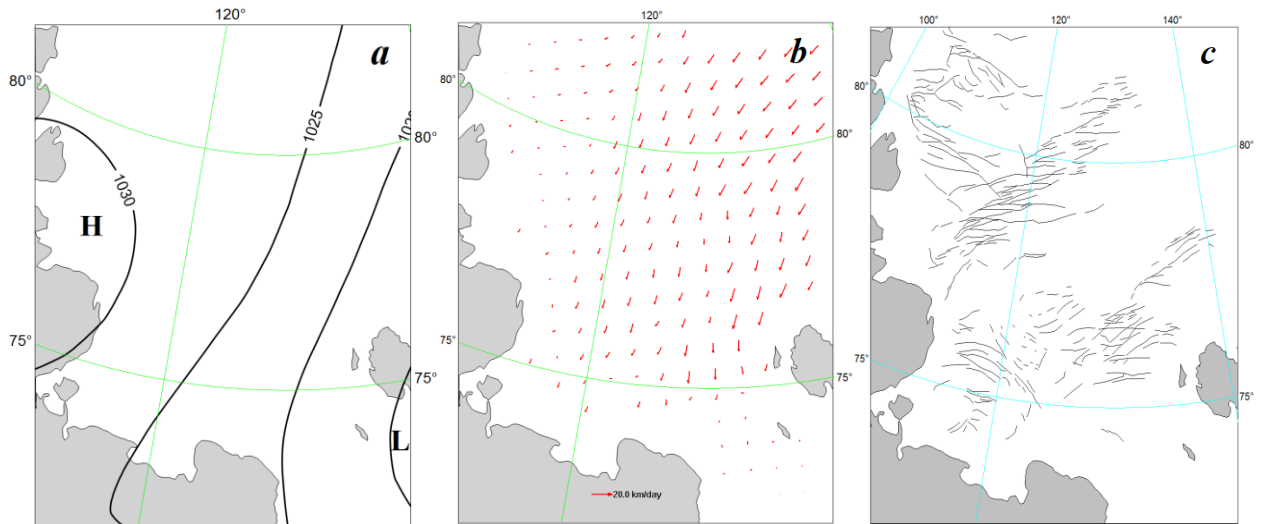


Fig.1. (a) Fields of atmospheric pressure, (b) drift and (c) leads in the ice cover in the Laptev Sea for 23.12.2017

At the changed drift direction the prevailing orientation of leads also changes. Fig.2 shows the atmospheric pressure, ice drift and leads in the ice cover for 29 December of 2017. The southwest air flow for this date was caused by the baric field presented in Fig. 2a. The atmospheric pressure gradient was large. The atmospheric pressure change within the sea was more than 20 mb, which determined a large speed of the southwest wind above the sea area and as result intensive ice drift. Fig. 2b shows the ice drift and Fig. 2c presents the field of leads corresponding to the drift. The number of leads for this date in the Laptev Sea is determined by a large non-uniformity of the ice drift velocity. The drift velocity increases both from the west to the east and from the south to the north of the sea. Correspondence between the ice drift direction in different parts of the sea is different. In the western part of the sea, there are cases where the orientation of leads differs significantly from the perpendicular one to the ice drift direction.

When the pressure gradient above the sea is insignificant, the ice drift velocity has small values or is close to zero. In this case, the system of leads formed by the previous synoptic situation is preserved. The drift can also be a result of other causes, for example, at sea level deflection at the time of onshore or offshore phenomena, contributing to formation of new leads.

Fig. 3a presents the baric situation above the Laptev Sea for 29 November of 2017. The atmospheric pressure change within the sea on this date was about 5 mb, and that is why the baric gradient was insignificant. The ice drift (Fig.3b) mainly had the north direction, although in the southeastern part of the sea the drift was in different directions and its velocity was not greater than 1 km/day. The direction of leads (Fig. 3c) over much of the sea area was close to a perpendicular one relative to the ice drift. The exception was the southeastern and western parts

of the sea, where the direction of leads is not in agreement with the ice drift. In the southeastern part, the cause was a weak drift. In the western part of the Laptev Sea near the coast, the leads are oriented along the drift flow. This is a peculiarity of the given region where leads are often located parallel to the coastline.

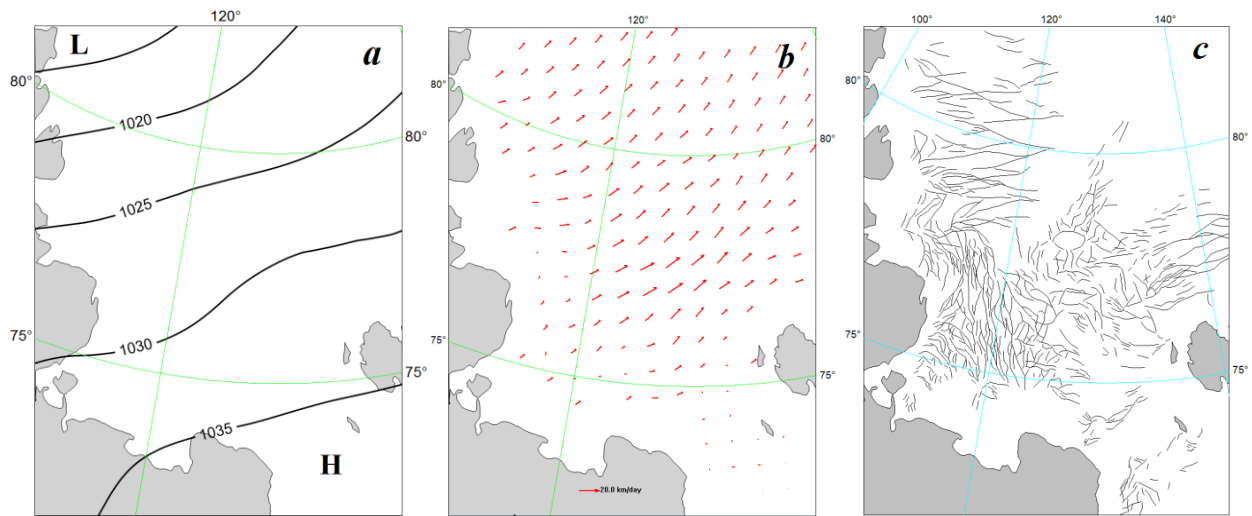


Fig.2. (a) Fields of atmospheric pressure, (b) drift and (c) leads in the ice cover in the Laptev Sea for 29.12.2017

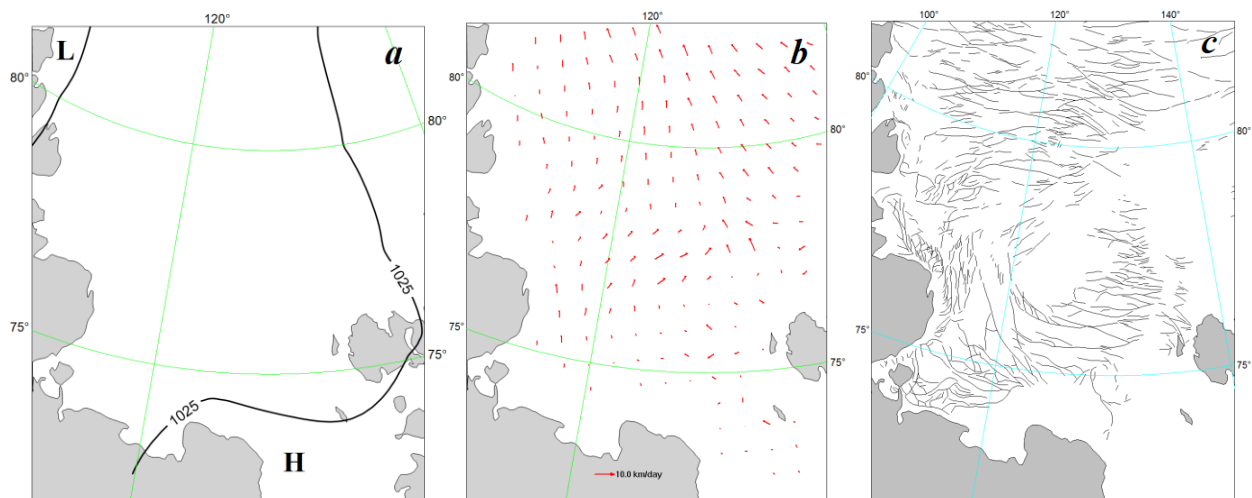


Fig.3. (a) Fields of atmospheric pressure, (b) drift and (c) leads in the ice cover in the Laptev Sea for 29.11.2017

An analysis of the presented figures shows that the existing dependence of the orientation of leads in the ice cover on the baric situation in the Laptev Sea is well manifested. In the case of the ice drift to the inner regions of the sea, the influence of the shore affects the orientation of leads. At the offshore drift from the sea, the influence of the shores is insignificant.

The use of natural connections between the atmospheric pressure fields, ice drift and divergence of its velocity and the characteristics of leads in the ice cover made it possible to develop an analogue method of forecasting of leads for the Arctic Basin (Gorbunov et al., 2008) and for the Kara Sea (Gorbunov et al., 2001). Taking into account a good skill score of this method it was decided to apply it for the development of the forecast of prevailing orientation and zones of formation of leads in such Arctic Seas as the Laptev and East-Siberian Seas.

The essence of the analogue method of the forecast is the choice by retrospective data of mean daily fields of atmospheric pressure, which within the study area have the maximum similarity by structure with the prognostic one. Such similarity suggests coincidence of the direction of air flows and the pressure gradient values and similarity in the location of zones of increased and decreased pressure and coincidence of the centers of cyclonic and anticyclonic circulation at their presence in the sea water area. If it is not possible to find in the electronic archive the atmospheric pressure field, which could be similar to the prognostic one within the entire sea, the choice of several fields each of them corresponding to the prognostic one only in one sea region is possible. In this case, the final analogue is formed as a mosaic of the chosen fields.

One adds to the actual surface atmospheric pressure field the calculated for the same date fields of ice drift and divergence of its velocity and also information on leads that existed in the ice cover during this period. A set of these data presents one complex analogue.

A set of complex analogues formed for the specific sea serves as a basis for issuing a forecast of the characteristics of leads by the considered method. At the successful selection of the analogue to the prognostic field of atmospheric pressure it is supposed that the characteristics of leads from this analogue will be similar to the prognostic ones. Based on the values of the drift velocity divergence in the analogue, zones of expected formation of leads (divergence is positive) and zones of compression (divergence is negative) will be determined (Losev et al., 2005). The high skill score of short-range forecasts of the fields of surface atmospheric pressure allows us to hope for the minimal number of unsatisfactory forecasts of characteristics of leads due to discrepancy of the prognostic pressure field to the real one.

GENERATION OF COMPLEX ANALOGUES

To have a possibility to quickly choose an analogue for the random atmospheric pressure field, it is necessary to divide and classify the different types of atmospheric circulation, observed in the study regions.

The existing typifications (Vangengeim, 1952, Girs, 1971, Dzerdzeevsky, 1975, Multanovsky, 1933) etc., covering vast spaces of the Arctic Seas and the continent are not suitable for our objective, as to forecast the leads it is important to fix the unidirectional air flows within one sea. As a basis for typification, 8 rhumbs of possible air flows during 24 hours were chosen. To each synoptic type, a number characterizing the air flow intensity was set for correspondence, which is determined by the number of isobars every 5 mb within the sea. Three more synoptic types are not connected with the unidirectional air flow. Two of them are characterized by the high or low pressure centers, and the third – dispersed baric field above the sea area.

The ice drift in the Arctic Seas is mainly a result of the wind impact on the ice cover surface. The ice drift field formed under the wind impact can change in the local sea areas due to the influence of the shores, seabed relief, level skewness and other factors. All these factors are taken into account to some extent in construction of the drift fields with the use of satellite observations of the motion of some ice floes. Information on the ice drift in the Arctic Seas is in free access on the Internet. In our work we have used Ocean and Sea Ice SAF (OSI SAF) data, belonging to EUMETSAT. These data present the components of the drift velocity vector calculated in the regular grid points with a step of 62,5 km. As a result of subsequent calculations of the ice drift velocity divergence (Volkov, 1971), a possibility appears to determine the zones of compression and divergence in the ice cover in the sea area.

The direction of leads in the ice cover can be determined by the direction of the small axis of the deformation tensor in the ice drift calculation (Losev and Gorbunov, 1998). However, the best result is obtained using the actual data on leads obtained from satellite images for the date corresponding to the analogue.

Satellite images are at present the main source of data on discontinuities in the ice cover. The main advantage of the space survey is a high range of vision, i.e. coverage by one image of the

surface of a large area. As the length of leads is often comparable with the sea size, for decoding of leads one uses images with low spatial resolution. Free TERRA/EOS-AM1 and SuomiNPP satellite images, available for performing ice cover surveys in the visible and infra-red ranges with a sufficiently wide scanning band have a spatial resolution of 250–500 m. The only drawback of satellite images obtained in the visible and IR-ranges is their dependence on meteorological conditions.

Decoding and digitizing of leads on satellite images is quite a labor-consuming process and is achieved by means of GIS ArcMap. In the course of processing of each image one records on it the geographical coordinates of the end of each relatively rectilinear segment of the lead, i.e., the lead segment within which its orientation does not visually change. The results are entered in to the electronic archive of leads, which serves as initial data for subsequent calculations of their characteristics.

As a characteristic of the prevailing direction of leads, their modal orientation is included to the analogue. For the Laptev and the East-Siberian Seas, this value is calculated for 100×100 km grid squares. The modal orientation of leads presents the direction at the range of $\pm 20^\circ$ from which the total length of leads in the square is the largest. The range itself is called modal at this. The ratio of the total length of the leads the orientation of which gets into the modal interval to the length of all leads in the square is called the exceedance probability of the modal interval. This value characterizes the degree of stability of the prevailing direction of leads and is expressed in percent. The presence of one more direction characterized by a sufficiently high value of exceedance probability (more than 25%) and differing from the modal orientation one by more than 30° suggests a bi-modal distribution of leads exists in the square. This suggests that two stable systems of leads exist in the square.

Fig.4 presents an example of the results of decoding and digitizing a SuomiNPP image to determine the characteristics of leads in the Laptev Sea. The modal direction of leads is presented in Fig.4c by the correspondingly oriented segments, the length of which is proportional to the exceedance probability of the modal interval. The specific length of leads characterizing their density of location in space is also calculated by the grid squares and is presented in Fig.4c by numerical values. This value reflects the total length of leads in the area of 1 km^2 and is expressed in meters per square kilometer.

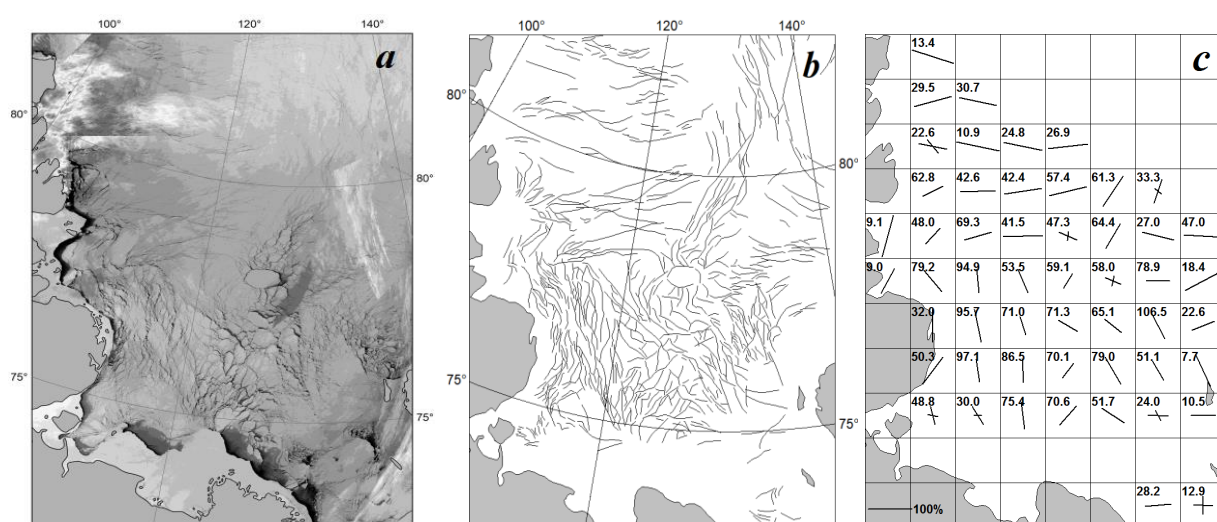


Fig.4. (a) Fragment of SuomiNPP satellite image of the Laptev Sea for 28 December of 2017, (b) result of digitizing leads data in the ice cover, (c) calculated prevailing orientation and specific length of leads

The fields of modal orientation and specific length of leads are a composite part of each analogue. During forecast preparation, one considers as priority data the data obtained from the electronic archive, i.e., actual data from the satellite image. In the case of absence of initial data on some squares due to impossibility of decoding the discontinuities as a result of cloudiness, one can use as their prevailing orientation the calculated direction of the small axis of the ellipse deformation rate (Losev et al., 2005).

If it is necessary to generalize the results of processing of images from different satellites and with different spatial resolution, an acute question appears about its influence on the obtained characteristics values of leads. The studies carried out allowed us to determine that the prevailing direction of leads in the ice cover is not the characteristic depending on the spatial resolution of the satellite image. The density of leads in space unlike their orientation significantly decreases in most cases with the decrease of the image spatial resolution. For example, the specific length of leads calculated from an image with a spatial resolution of 500 m is on average by 1,7–1,9 times smaller compared with an image having a spatial resolution of 250 m. The results of the analysis of data on leads decoded from the images with a spatial resolution of 250 m, 375 m and 500 m allowed us to approximate the relationship between the specific length values of leads obtained from processing of images with a different spatial resolution. The derived regression equations make it possible to make a reliable estimates of the density of leads in lower resolution images.

Thus, creation of one complex analogue requires actual data on the surface atmospheric pressure, ice drift and location of leads in the sea ice cover. In the course of processing of initial information there are calculated the fields of atmospheric pressure, ice drift velocity vectors, fields of its velocity divergence, modal orientation and specific length of leads in the ice cover. The result of visualization of the fully formed complex analogue is presented in Fig.5.

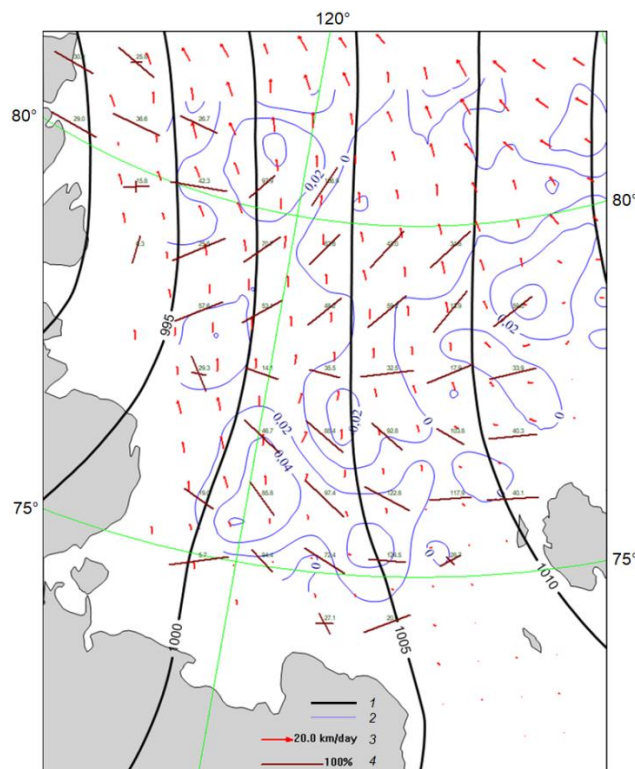


Fig. 5. Complex analogue for the Laptev Sea for 27 February of 2019

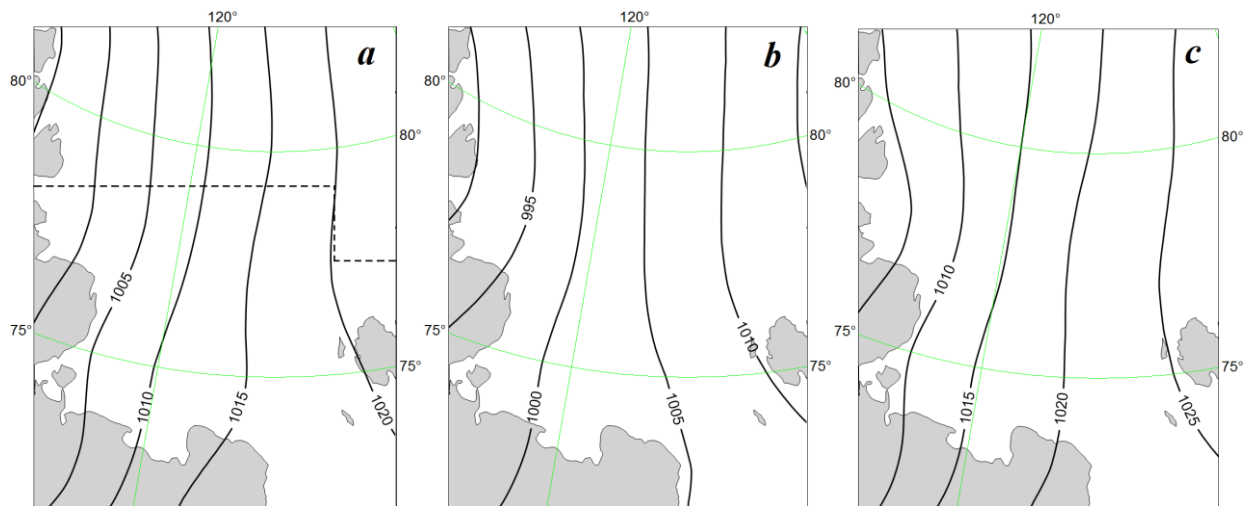
1 – Isobars; 2 – isolines of drift velocity divergence; 3 – ice drift vectors, the length of which is proportional to its velocity module; 4 – modal orientation of leads, where the segment length is proportional to exceedance probability of the modal interval

DIAGNOSTIC FORECASTS

By the present time several dozens of complex analogues were formed for the Laptev Sea, resulting in preparation of two diagnostic forecasts. The corresponding field of one of the available analogues was assumed to be a prognostic field of atmospheric pressure, while the other served as data for making the forecast. In the first case the prognostic field referred by the adopted typification type S6 (south direction of the air flows with intensity of six isobars with a step of 5 mb), and in the second – SW4 (southwest direction of air flows with intensity of four isobars with a step of 5mb).

In the first forecast for the baric field assumed to be prognostic, it was possible to choose two analogues.

Fig.6 presents the atmospheric pressure fields with one of them assumed to be prognostic (Fig.6a), and two other fields were analogues to it. The field in Fig.6b was assumed to be an analogue for the northern part of the sea and in Fig. 6c – for the southern. The division when using the first and the second analogues is shown in Fig.6a by a dashed line.



In order to assess the skill score of the forecast of location of zones with formation of leads, squares were identified on the chosen analogues, in which the highest values of the specific length of leads were noted. In the analogue for the northern part of the sea, four such squares were selected and in the analogue for the southern part – five. All these squares were located in the zones with positive values of divergence of ice drift velocity, and the specific length of leads in them exceeded on average by 3.5 times the corresponding value in the rest of the squares. The regions with negative divergence values on the selected analogues were absent.

The specific length values in the identified squares were compared with the corresponding values in the analogue assumed to be prognostic. Of nine analyzed squares, data on leads in the prognostic analogue were absent in one, in five, the specific length values were overestimated, in one – underestimated and in two squares, its value did not differ significantly from the values in the adjoining squares. Of three local areas with the positive values of divergence of ice drift velocity, identified on the chosen analogues, two preserved its location also on the prognostic one. Thus, the exceedance probability of the forecast of the zone of formation of leads can be considered equal to 66%.

When preparing the second diagnostic forecast it was required to use three analogues. Of 34 squares provided with actual data on leads, on the selected analogues the permissible value of 30° was not exceeded in 25 squares, i.e., in 74% of the cases. In each of three selected analogues in the zones of correspondence of their atmospheric pressure field to the prognostic one there was present by one local area with the positive values of divergence of ice drift velocity. All three areas were fixed on the prognostic analogue, although the sizes of zones slightly differed. Unfortunately, data on leads were available only in one of them, but the values of the specific length of leads within it were higher than in the adjoining squares.

CONCLUSIONS

The main factor causing the ice drift in the Arctic Seas and as a result formation of leads is the wind. In the case of wind absence, leads in the ice cover can be formed under the action of the drift, caused by other factors, for example, level deflection.

The primary predictor for forecasting the characteristics of leads is the atmospheric pressure field, on which wind depends and as a result, the ice drift. At the change of the wind direction the modification of the system of leads in the ice cover occurs. The direction of leads and their number change within several hours after the change of air flow direction.

In the Laptev Sea, data on which were used in the article, allow us to make a conclusion that the influence of the shores on the ice drift and as a result on the processes of formation of leads is quite significant. The largest influence of the shores is produced on the drift with the direction to the south into the inner sea regions. At the ice export from the sea the influence of the shores on the drift is not so significant.

Digitizing of satellite images of low resolution allows us to obtain the length and orientation relative to rectilinear segments of leads, which makes it possible to calculate such their characteristics as the modal orientation and the specific length.

Prepared by the method of analogues diagnostic forecasts of modal orientation of leads and zones of their formation in the Laptev Sea yielded good results. This allows us to hope that the analogue forecasting method will give satisfactory results in the other Arctic Seas as well.

Further increase of the number of complex analogues will decrease the probability of encountering a similar baric field as a prognostic one.

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(cryosphere), processes of ice interaction with natural objects and engineering structures for the Arctic and technologies of hydrometeorological provision of users”.

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