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Numerical modeling of elastic and thermal dynamical processes in artificial ice island

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ABSTRACT

This article is dedicated to sustainability of an artificial ice island to mechanical and thermal interactions with use of numerical modelling. Mechanical impacts, which are caused by the shock impacts during drilling, as well as the pressure from man-made structures on the island are modeled by solving continuum mechanics equations numerically using grid-characteristic method. Grid-characteristic method is the most precise to describe dynamic processes in seismic tomography because it considers the origin of seismic waves phenomena. Numerical method allows to develop computational algorithms which are correct on the boundaries and interfaces of the integrational domain. Stress distribution along with elastic wave propagation is studies. Von Mises criterion is used to analyze the sustainability of the island to fractures.

Stefan task is formulated to analyze phase change due to thermal impacts on the island. Enthalpy method is developed, it's limits of applicability are discussed. Pismen-Rekford scheme, which is absolutely stable in two-dimensional case, is presented to solve the problem effectively with use of arbitrary time step. Developed method is capable of considering special effects i.e. water flow and melted water runoff from the top of the island which is important for the task. Modeling also allows different substances in the medium with arbitrary initial and boundary conditions, environment conditions, heat flux from solar radiation, water currents. Using this approach field of temperature inside an ice island after construction is calculated. Research of the resistance to the seasonal temperature changes of the island is conducted.

KEY WORDS: Ice island; Mathematical modelling; Grid-characteristics method; Stefan task.

INTRODUCTION

Modern problems of developing of the Arctic region are demanding new requirements for facilities on the shelf. Artificial ice islands are cheap and ecological alternative to ordinary boring equipment for developing gas and oil deposits in the harsh Arctic environment. This approach has been successfully implemented in Canada, as it is shown in (Crawford et al., 2018). In the work (Petrov, 2019) it is noted that safety of the personnel and constructions on those islands are problems of current interest due to probability of its failure because of drilling or high static loads of the buildings.

In modern software finite-element method is used to study sustainability of those structures, as shown in (Xunqiang et al., 2013; Nikolic et al., 2017). Finite difference methods studied in (Moczo, 2007), spectral element methods regarded in (Komatitsch and Tromp, 1999), discontinuous Galerkin methods are used to model seismic wave propagation, as shown in works (Wilcox et al., 2017) and (De Basabe et al., 2008). Grid-characteristic method with interpolation on rectangular and hexahedral structured meshes and triangular and tetrahedral unstructured meshes, studied in (Favorskaya et al., 2019), are used in this paper. This approach has shown good applicability for seismic exploration in fractured media, which was proved in (Petrov et al., 2019).

Alongside with mechanical sustainability it is necessary to consider thermal impacts (melting of the ice island). It is necessary to know distribution of temperature inside the island for correct solution of the mechanical sustainability problem, both for dynamical and static loads. Also, it is important to study its durability to seasonal thermal changes. Approaches to account for environmental effects are considered in works (Buzin et al., 2009; Comfort et al, 2013).

In this paper a way to numerically model ice island melting, which is based on the solution of the partial derivative equation describing evolution of the system with multiple phases of the same substance and moving boundary between them or in other words the Stefan problem, is presented. This task has been reviewed in several works including (Biryukov et al., 2017; Jonsson, 2013; White, 1982). Numerical solution of the Stefan task is examined in (Budak et al., 1965; Dar'in et al., 1987). In works (Vasil'ev F.P., 1968; Bachelis, 1969), the method of straight lines was considered. Another popular method is finite element method and finite difference methods.

Enthalpy method, which was previously regarded in (Buchko) is used in this work. For this method the task is reformulated in terms of heat content. Designed method is also capable of modeling water flow, melted water runoff from the top of the island and other important phenomena. Distribution of temperature inside the island is calculated using this method. Its sustainability to seasonal temperature changes is studied.

MATHEMATICAL MODEL

Elastic processes

Governing system of equations for linear elastic medium can be represented with the following equations:

$$\rho \frac{\partial V_i}{\partial t} = \frac{\partial T_{ji}}{\partial x_j}, \ \frac{\partial \mathrm{T_{ij}}}{\partial \mathrm{t}} = \lambda \left(\sum_k \frac{\partial V_k}{\partial x_k} \right) I_{ij} + \mu \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \tag{1}$$

Where V_i are the velocity components, $\mathrm{T_{ij}}$ are stress tensor components, x_i are radius-vector components, I_{ij} is a unite tensor, t is time, ρ is density of the material, λ and μ are its Lamé parameters. Introducing $\vec{u} = \{V_x, V_y, V_z, T_{xx}, T_{yy}, T_{zz}, T_{xy}, T_{xz}, T_{yz}\}$ reduces system to the form:

$$\frac{\partial \vec{u}}{\partial t} + \sum_{i=1,2,3} A_i \frac{\partial \vec{u}}{\partial \xi_i} = 0 \tag{2}$$

Numerical solution could be obtained with the use of grid-characteristics method, which was reviewed in works (Favorskaya et al., 2019; Petrov et al. 2019). After coordinate splitting, change of variables the system is reduced to the system of independent scalar transport equations for Riemann invariants

$$\frac{\partial \vec{w}}{\partial t} + \Omega_i \frac{\partial \vec{w}}{\partial \xi'_i} = 0, i = 1,2,3 \quad (3)$$

The vector $\vec{w}$ is transferred to the new time layer using the formula

$$w_k^{n+1}(\xi'_i) = w_k^n(\xi'_i - \omega_k \tau) \quad (4)$$

Where τ is the time step, n is the previous step, $n + 1$ is the current step.

After all quantities have been transferred the transition to the $\vec{u}$, the sought values vector, occurs.

Values at each point are then computed using the values at reference mesh points $\vec{u}(\vec{r}_{ijkl})$ and weights of those points $p_{ijkl}(\vec{r})$ according to the formula

$$\vec{u}(\vec{r}) = \sum_{i,j,k,l} p_{ijkl}(\vec{r}) \vec{u}(\vec{r}_{ijkl}) \quad (5)$$

Thermal dynamical processes

The first law of thermodynamics in terms of enthalpy (H) or thermal content may be written down in the following form:

$$\delta Q = dH - VdP \quad (6)$$

Where δQ is transferred heat, V is the volume, dP is the change of pressure. For slow equilibrium processes with constant pressure the equation will take the following form

$$\delta Q = dH \quad (7)$$

Enthalpy is a state function and is certainly defined by temperature for known phase. Picking small volume inside our computational domain we can rewrite previous equation

$$-\int_{S=\partial V} \vec{q} \cdot d\vec{S} = \int_V \frac{\partial H}{\partial t} dV \quad (8)$$

Where $\vec{q}$ is heat flux. Using divergence theorem, we could get continuity equation for heat.

$$div(\vec{q}) + \frac{\partial H}{\partial t} = 0 \quad (9)$$

Heat flux is defined by gradient of temperature and thermal conductivity k, which is a property of the specific phase. It could be calculated with the use of Fourier's law for heat conduction.

$$\vec{V}\left(-k(H, x, y, z)\vec{V}T\right) + \frac{\partial H}{\partial t} = 0 \quad (10)$$

Enthalpy or heat content are defined for a specific substance with the following equation

$$Q = \begin{cases} \rho_S C_S T, & T < T_0 \\ \rho_L C_L (T - T_0) + \rho_S C_S T_p + \rho_S \lambda, & T > T_0 \end{cases} \quad (11)$$

Where ρ_S and C_S – density and specific heat capacity of solid state, ρ_L and C_L – of liquid state, T_0 – the phase transition temperature, λ – specific heat of fusion. Inverse transition is possible using the following equation.

$$T = \begin{cases} Q \cdot \rho_S^{-1} C_S^{-1}, & Q < \rho_S C_S T_s = Q_- \\ T_0\,, & Q_- < Q < Q_+ \\ \frac{Q + (\rho_L C_L - \rho_S C_S) T_0 - \rho_S \lambda}{\rho_L C_L}, & Q > \rho_S C_S T_s + \rho_S \lambda = Q_+ \end{cases} \tag{12}$$

where Q_- and Q_+ are limits of the value of heat content at the phase transition temperature. It worth mentioning, that equation (11) is discontinuous, because substance at its phase transition temperature can be in either of two phases. To get the final hyperbolic quasilinear differential equation in partial derivatives we will merge (10, 11, 12) an get

$$\frac{\partial}{\partial x}\left(k(Q)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(Q)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(Q)\frac{\partial T}{\partial z}\right) = \frac{\partial H}{\partial t} \tag{13}$$

In this equation temperature and thermal conductivity functions can be different in different parts of the computational domain. It allows to correctly consider different substances.

To numerically solve equation (13) Pismen-Rekford numerical scheme was used. Its key features and implementation are described in (Albu et al., 2007; Albu et al., 2011). It showed the best speed of all compared methods.

PROBLEM FORMULATION

Computational domain of the computed problems is an ice island with height of 10 meters and horizontal length of 300 meters. Model of an ice island was studied in (Petrov, 2020). Ice island is submerged into the water by 8 meters and lies on solid ground. Layout diagram can be seen on figure 1. Ice, water and ground are homogeneous substances. Elastic characteristics are shown in table 1. Thermophysical properties are shown in table 2. Assume that ice melts at $T_0 = 0°\,\mathrm{C}$ and its specific heat of fusion $\lambda = 334\ \mathrm{kJ/kg}$.

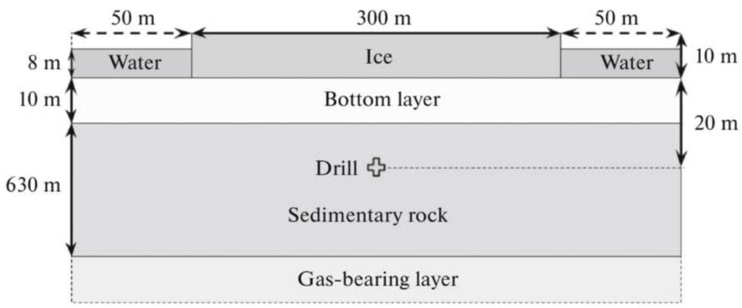


Figure 1. Schematic view of the computational domain

Table 1. Elastic characteristics of the substances

Substance	P-wave velocity, $\frac{m}{s}$	S-wave velocity, $\frac{m}{s}$	Density, $\frac{kg}{m^3}$
Ice	3940	2493	917

Water	1500	–	1025
Bottom layer	1806	316	2000
Sedimentary rock	2250	1000	2000

Table 2. Thermophysical characteristics of the substances

Substance	Density, $\frac{kg}{m^3}$	Thermal conductivity, $\frac{W}{m \cdot K}$	Specific heat capacity, $\frac{J}{kg \cdot K}$
Ice	917	0.591	2100
Water	1000	2.22	4180
Bottom layer	2500	0.8	750
Air	1.60	0.022	–

We have modelled two types of loads: static load from buildings and other constructions on the island and shock load from the drill. Temperature distribution was computed inside the island with average winter air temperature. Computation of the life span of the island were made along with estimation of its resistance to seasonal temperature changes. Three main models were considered: ice island without water flow and constant ground temperature, ice island with water flow and constant ground temperature, ice island with both water flow and freezing seabed.

RESULTS

Impact load of the drill

Wave propagation during drilling was studied, stress distribution inside ice island along with its sustainability to loads were studied. Figure 2 shows Von Mises stress distribution at time $t = 0.032\ s$ after the start of the drilling.

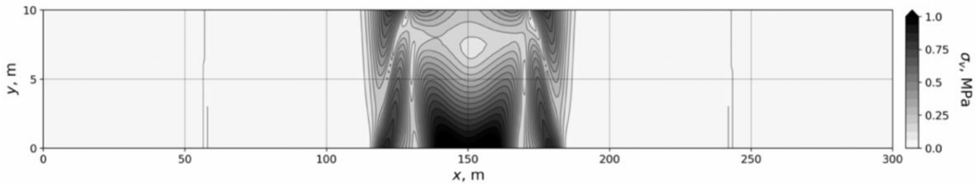


Figure 2. Von Mises stress distribution under drilling impact load

It was empirically found that ice island starts to break at wave amplitudes from the drill which are around $10^{12} Pa$. This value is too big and can not be reached in real conditions.

Static load

Static load distribution was found by applying relaxation method, as reviewed in (Fedorenko, 1962). Stress distribution was regarded as stable if velocity of magnitude for elastic wave propagation was 20-30 times lower than initial velocity. After 200 thousand time steps the velocity magnitude was reduced roughly by a factor of 33. Thus, the resulting stress distribution can be regarded as stable. Figure 3 shows von Mises stress distribution for a stable state.

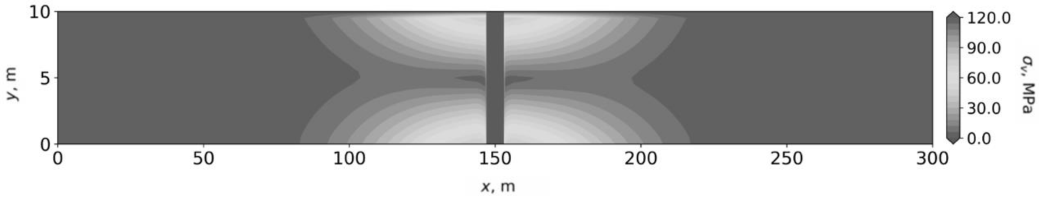


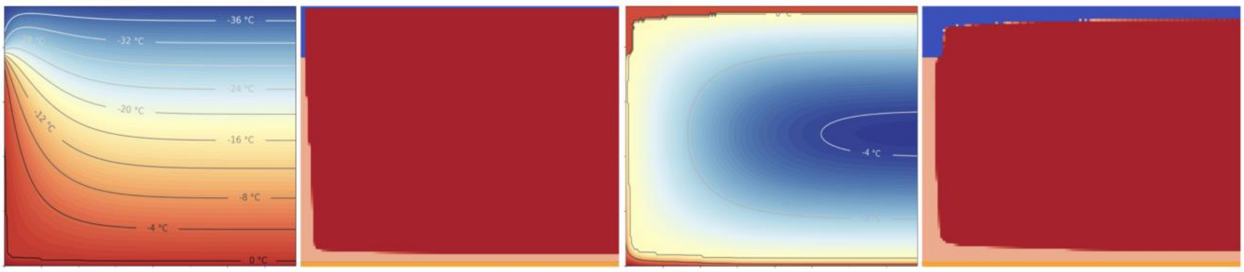
Figure 3. Von Mises stress distribution under static load

Maximum Von Mises stress in the problem was 2.2 kPa. This value is less than the limiting shear stress for ice. Thus, at real values of the static load, the ice does not break.

Ice island evolution without water flow and constant ground temperature

Numerical experiments were carried out to determine the equilibrium temperature distribution in the ice island at an air temperature of -40° C, a bottom soil temperature of 5° C and a water temperature of 3° C. The problem of establishing was solved, the initial temperature of the ice island was -10° C, the contacting substances (water, air, soil) were set in the form of boundary conditions with given temperatures, heat capacities and thermal conductivity coefficients.

Results of the experiments can be seen on figure 4 (*a, b*). A period of 150 days was modeled. On the left part of the figure temperature distribution can be seen. Map of substances and phases can be seen on the right. On this map air is represented with blue, ice is red, water is cream-colored, seabed is yellow.



a *b* *c* *d*

Figure 4. Temperature distribution inside an ice island (*a, c*) and a phase map (*b, d*) after construction (*a, b*) and after 150 days of extreme air temperature (*c, d*) in the task without water flow and constant ground temperature

After that a study was made to test the stability of the ice island under seasonal temperature changes. The air temperature has been changed to 3° C. The resulting ice island after 150 days is presented in figure 4 (*c, d*). It can be seen that the ice island practically did not melt near its side since we did not consider the flow of water a temperature gradient established there. Top of the island melted significantly since after melting, the water immediately runoff down and the boundary condition was set on the remaining ice. Even a relatively low coefficient of thermal conductivity of the air provided a serious heat flow, which melted the island.

Ice island evolution with water flow and constant ground temperature

We have seen that without the water flow the ice island will not melt significantly. To consider water flow a simple technique was developed. For any melted water with a temperature strictly greater than zero at the end of the time step, we explicitly set the temperature equal to 3° C. We use the results obtained in the previous computations and similarly to the previous calculation, the results after 150 days of extreme outside temperature are shown in Figure 5. It could be clearly seen that the island has melted on the bottom. This happened due to the fact that bottom edge has a positive constant temperature a thin layer of water appeared between the edge and the island. Further, the constant water temperature created a large temperature difference with the ice and, as a result, an abnormally high heat flux through the lower edge.

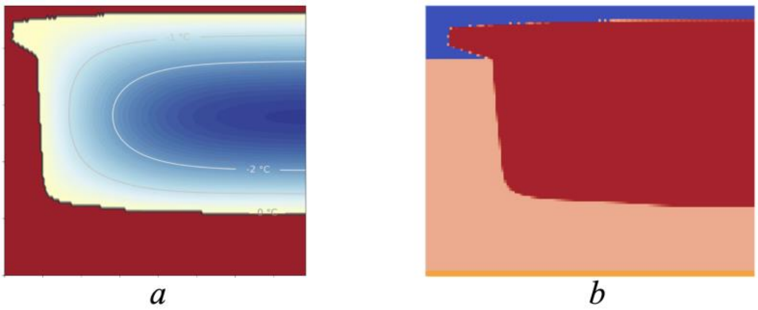


Figure 6. Temperature distribution inside an ice island (*a*) and a phase map (*b*) after 150 days of extreme air temperature in the task with water flow and constant ground temperature

Ice island evolution with water flow and freezing seabed

In order to solve the problem that arose in the previous numerical experiments, it is possible to change the bottom boundary condition and instead add a bottom soil to computational domain. To do this, we use a integration array of 300 meters by 20 meters, where ice occupies the upper half, similar to the previous tasks, and 10 meters below are filled with seabed substance. The initial temperature distribution computation is similar to the previous experiments, but the soil has an initial temperature of +5° C. The boundary conditions for water and air are similar to the previous tasks with the addition of zero heat flux through the bottom and side edges of the soil.

The results of modeling are shown in Figure 7 (*a, b*). It can be seen that the characteristic freezing depth is about five meters. Right side of Figure 6 is a map of phase states in the computational domain. On this map red is ice, dark blue is air, blue is water and a solid seabed border is denoted with a black line.

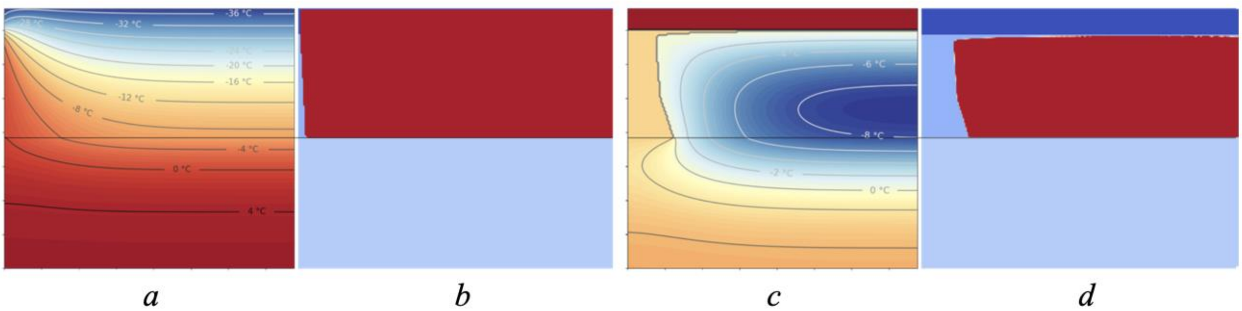


Figure 7. Temperature distribution inside an ice island (*a, c*) and a phase map (*b, d*) after

construction (*a, b*) and after 150 days of extreme air temperature (*c, d*) in the task with water flow and freezing seabed

For this temperature distribution, the problem of stability in extreme conditions with the flow of water was modeled. The air temperature was set to +10 ° C. Experiment results are shown in Figure 7 (*c, d*). It can be seen that after 150 days the topside melted completely. The left and right edge of the island has also melted seriously. The temperature in the soil has practically not changed.

CONCLUSIONS

In this article numerical experiments to test artificial ice island sustainability to both static and dynamical mechanical loads and resistance to melting are discussed. This research showed that reviewed ice island configuration is viable and durable.

The approach based on grid-characteristic method, used in this work allows modeling of ice island stability problems with simulation of drill impact and exposure to static load. The approach can help estimate influence of different impacts on ice island and determine their critical value which lead to island destruction. These results can be used in research and explorational works on ice island.

Thermodynamic processes in the ice medium inside of the ice island are modeled using the enthalpy method. The modification allows one to consider the mixing of water. For a two-dimensional island of a rectangular shape, the distribution after a long period of time is calculated in various cases. Numerical experiments allow to prove the seasonality of the construction of such an island and suggest possible procedures for auditing its integrity and extending its life span.

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