The effect of ice fragment shape on model-scale brash ice material properties for ship model testing

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ABSTRACT

Safe and efficient shipping in ice-covered seas requires ships to have a good ice performance. According to Finnish Swedish Ice Class Rules, the ship performance for ice classes can be verified through brash ice channel model tests. However, the current regulations do not specify the brash ice condition for model testing in detail. It requires more understanding of model brash ice material properties and behavior to develop test procedures for providing standardized outcomes.

This paper aims to bring new insight into the ice fragment shape effect on brash ice material properties. It first overviews the effect of particle shapes on granular material properties and behavior from ice mechanics and soil mechanics. This paper then proposes a preliminary approach to evaluate model brash ice fragment shape from digital images and attempts to assess shape characters and material properties influenced by the repeated ship model testing.

KEY WORDS Brash ice channel; Ice fragment shape; Model test.

1. INTRODUCTION

Safe winter navigation requires ships with sufficient ice-going capability. The current winter navigation system in the Baltic Sea is based on ice class classification of merchant ships. For granting an ice class, the Finnish-Swedish Ice Class Rules provide two methods for determining the ship performance: empirical calculation formulas and ice model tests. As the brash ice channel is a typical ice condition for merchant ships, the ice model tests to entitle an ice-class for ships are required to be conducted in defined brash ice channels.

Brash ice is commonly understood as a floating accumulation of round ice fragments or pieces in nature, as shown in Figure 1. Brash ice in channel has many properties which are neither easily described nor measured. As the ship model testing performance in the brash ice channel can be used to grant the ship's ice class, there is a need to standardize the brash ice material properties in model scale to ensure harmonized test results.

Many approaches have been applied in brash ice research for decades, including measurements in full scale (e.g., Tuovinen, 1979; Sandkvist, 1981; Nortala-Hoikkanen, 1997; Bonath et al., 2019), laboratory measurements in model scale (e.g., Keinonen and Nyman, 1978; Fransson and Sandkvist, 1985; Bruneau et al.,1996; Bridges et al. 2019), using different numerical methods (e.g., Liferov, 2005; Polojärvi et al., 2015) and analytical methods (Keinonen, 1979; Wilhelmson, 1996), and combining these different approaches to observe and understand better the brash ice behavior (e.g., Liferov and Bonnemaire, 2005). Regarding the full-scale and laboratory measurement, the full-scale measurements of brash ice properties are expensive, and they involve uncontrollable variables. In contrast, model testing facilities can provide reasonable control of variables in laboratory conditions. However, it still requires considerable understanding of the brash ice material properties and behavior to correctly model the ice in nature and ensure accurate and reliable test outcomes.

The brash ice fragment shape as an essential character has been mentioned in multiple sources (Sandkvist, 1981; Sandkvist,1986; Nortala-Hoikkanen & Soininen, 1990; Gong et al., 2019). As a result of several ship passages, the shape of brash ice fragments gradually become less angular and solid due to repeated freezing cycles, as the ice fragments are spinning in the water. However, with current model test procedures, correctly modeled ice fragment shape is not easily achieved. Thus, there is an interest to define what is the fragment shape, and does it influence on ship performance in ice model tests. Thus, this paper focuses on characterizing model-scale brash ice fragment shape and assessing the shape effect on brash ice material properties. The paper is organized in two parts, starting with an overview of granular soil particle shape, and measuring procedures and particle shape influence on material properties. After the overview, this paper proposes a simple method for characterizing model brash ice shape using computational geometric methods. Lastly, it presents a preliminary analysis of the effect of model brash ice shape on the material properties by repeated ship model testing.

Figure 1. A typical old channel in the Bay of Bothnia, 2021. Brash ice fragments in the channel become less angular and more solid as a results of multiple ship passages.

2. PARTICLE SHAPE DESCRIPTION

Granular material can be described by particle, or fragment, size and shape. Both have significant roles in material properties (Cabalar, 2018). The particle size and distribution of a granular material have been given much attention due to practical reasons; for instance, the particle size distribution can easily be controlled by sieving and described by the measured dimensions. By comparison, describing and controlling the shape requires more complex approaches (Rodriguez, 2013).

In soil mechanics, the particle shape has been described using qualitative or quantitative approaches. Traditionally, the qualitative approaches describe the shape with simple descriptive words, such as “spherical” or “flaky”. The quantitative description is based on geometrical measures and the qualitative description. The quantitative method is reproducible and therefore preferred by field engineering. However, both descriptions are experience-based approaches.

The particle shape determination techniques have developed further from traditional measurements by hand, comparison charts and sieving analysis. As the technology developing, the particle shape can be described in 2 or 3 dimensions (2D or 3D) using sophisticated equipment to combine information from different views for numerical rebuilding particle models (Rodriguez, 2013). As narrated by Rodriguez (2013), the fragment shape can be described in three scales: large scale revealing morphology, intermediate scale describing irregularities, and small scale focusing on surface texture. On the intermediate scale, the fragment shape can also be characterized by sphericity (circularity) and roundness (angularity), among others. However, it notices that the terminology of shape characters has not been standardized among geology, geotechnical engineering, mathematics, and computer science. For reference, some mathematical approaches of characterizing the fragment shape have been introduced in standards (e.g., ASTM D5821, EN 933-3 and BS 812).

Compared with the soil, brash ice has unique granular material characters, such as relatively weak ice material strength and temperature dependency. These material characters limit the direct application of geotechnical techniques for measuring brash ice shape.

3. EFFECT OF FRAGMENT SHAPE ON GRANURAL MATERIAL PROPERTIES

The section outlines the current understanding of fragment shape influence on granular material properties. Most studies introduced here relate to soil mechanics. All findings cannot be directly extrapolated to brash ice, but it reviews the links between granular material fragments shape and material fragment interaction. The material properties discussed here are based on earlier research (Matala, 2021), including porosity, internal friction angle, angle of repose and compressibility, which were used to assess the fragment strength effect.

The fragment shape can be characterized by sphericity and roundness. The definition of sphericity and roundness can refer to Cruz-Matías et al. (2019). *Sphericity* (*circularity*) is a measure of the degree to which an object approximates a sphere shape. *Roundness* (*angularity*) is the measure of the sharpness of object edges and corners, such as angular protrusions (convexities) and indentations (concavities).

Porosity

Several studies have reported a link between the particle shape (sphericity and roundness) and porosity (void ratio). In general, the volume of the void increases with decreasing sphericity, resulting from particle rearrangement, and interlocking as the irregularities limit the particle mobility and ability to settle in dense configurations (Cho et al. 2006; Rodriguez, 2013). The roundness of the particles was also observed to affect the void ratio. For example, the maximum void ratio e_{max} would decrease with increases in roundness R: $e_{max} = 1.3 - 0.62R$ (Cho et al. 2006). This empirical formula was obtained from the experiments of sand and glass bead mixtures.

Internal friction angle

Internal friction angle is dependent on particle shape (Rodriguez, 2013 and Cho et al., 2006). A higher particle irregularity increases the internal friction angle on the critical stage. A possible explanation is that particle irregularity limits particle mobility. For instance, Cabalar (2018) showed that the crushed sand with angular-shaped fragments had higher shear stress than the sand with round particles. The experimental results of Yang and Luo (2015) indicated that the particle shape significantly alters the shearing resistance of granular soils; correspondingly, the internal friction angle increases with decreasing roundness. Mirnyy and Merkin (2016) state that the particle shape affects the particle interaction. This mechanism may result in different internal friction angles between fine- and coarse-grained (granular) materials. The particle interaction of coarse-grained materials involves rolling the particles, in which surface roughness and roundness are significant factors, while the sliding friction is prominent for fine-grained soils. Multiple researchers have reported empirical relations between internal friction angle φ and roundness R in granular soil. For example, Cho et al. (2006) proposes $\varphi = 42 - 17\,R$.

Regarding ice research, the ice fragment shape affects the initial void ratio-depended friction angle, as observed by Liferov and B. Bonnemaire (2005). In nature, brash ice fragments are considered strong and solid. The effect of soil fragment shape on the internal friction angle presented above could be applied to nature brash ice. However, the model brash ice scaled by Froude-scaling rules is often too soft to keep the ice fragment shape after the ship model tests.

Angle of repose

The angle of repose α has found to decrease with increasing roundness in soils by experimental (Rodriguez, 2013) and numerical methods (Wei et al., 2019). Rodriguez (2013) presents a formula: $\alpha = 41.7 - 14.4\,R$.

Interestingly, Friedman and Robinson (2002) proposed using the angle of repose as an indicator of particle shape, which could be a feasible method for ice model testing.

Compressibility

Regarding brash ice, compressibility was investigated by observing how easily water escapes from model brash ice sample through displacing the brash ice sample by applying an external force, as addressed in Matala (2021). The water escape requires particle rearrangement, which relates to the shape-depended particle mobility.

The current knowledge in soil mechanics and ice mechanics indicates that brash ice fragment shape significantly affects the brash ice material properties. Furthermore, the brash ice fragment shape influences brash ice fragment interaction and further the brash ice pile-up and dynamics around a moving ship hull. Therefore, it requires more understanding of brash ice shape characterization for standardization of ship model tests in the brash ice channel.

4. EVALUATION OF MODEL BRASH ICE SHAPE

The previous overview presents the effect of granular material fragment shape on material properties. Since brash ice is considered one type of granular material, this paper proposes a simple approach of characterizing model brash ice shape by digital images to study the shape effect on material properties. Three comparative sets of model brash ice are evaluated in terms of the detected number and size of ice fragments and the shape characters regarding sphericity (circularity) and roundness (angularity).

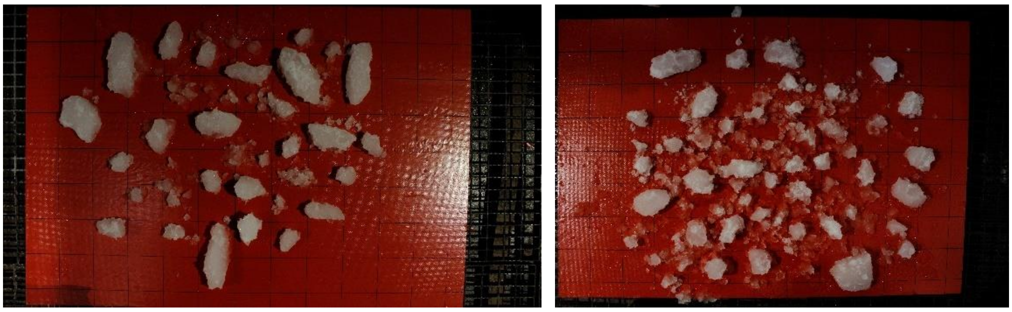
Two types of model brash ice for ship performance tests in channel are studied. Following the model testing procedures often used in model testing facilities, brash ice in the channel can be made of ice fragments manually cut from level ice. The brash ice sample is collected before the ship performance test, which is denoted as "new channel" in this paper. After the ship performance test, the brash ice is recycled and rearranged in the channel for the following ship performance test. Such brash ice channel is denoted as "old channel" as the brash ice is considered to lose the initial material properties after the ship test potentially. This type of brash ice channel is prepared again as "Old channel 2" for the last channel test.

In this paper, nine brash ice samples are studied, made from one 0.8 m thick level ice sheet with flexural strength σ_f corresponding to 500 kPa and two 0.8 m level ice sheets with σ_f =1000 kPa in full scale. The higher σ_f was chosen to further study the scaling effect on the model ice flexural strength. Two sheets with a high σ_f were used for validating the test repeatability. The samples were collected using a box (36 cm × 26 cm × 14 cm) from the brash ice channel prepared for a ship model test. Sample numbers and level ice parameters are listed in Table 1. The scaling factor was 20 by considering the ship model scale. The target channel thickness was 1.3 m (65 mm in model scale), and the channel width was two ship's beams. The brash ice sample collecting longitudinal location in the basin and collecting volume were the same in all channel cases.

Table 1: Brash ice sample number (in grey background) and characterization

Brash ice sample number	Level ice flexural strength σ_f in full scale [kPa] / **in model scale [kPa]**		
	Ice sheet 1 500 / **25**	Ice sheet 2 1000 / **50**	Ice sheet 3 1000 / **50**
New channel	11	21	31
Old channel 1	12	22	32
Old channel 2	13	23	33

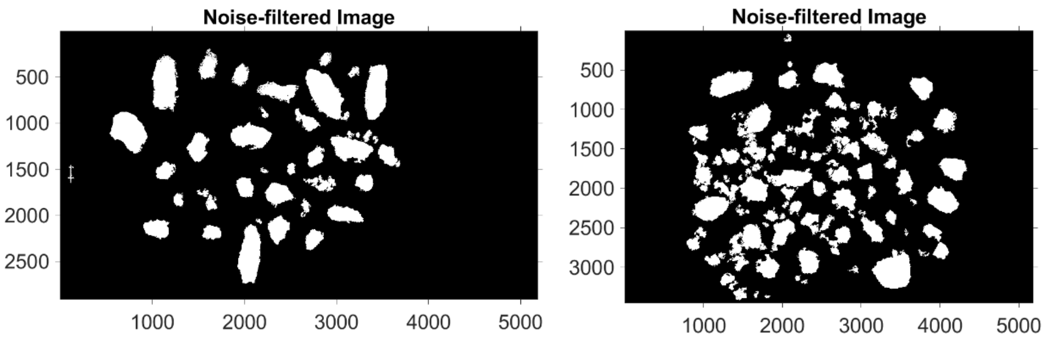
The brash ice sample was displayed on a red grid panel and photo recorded. Figure 3 shows the brash ice sample 11 and 13. Matlab Image Processing Toolbox is used to post-process the ice shape in images (Mathworks, 2021). The sample image is converted into noise filtered Black & White binary image with adjustment of overexposures.



(a) Sample 11 (b) Sample 13

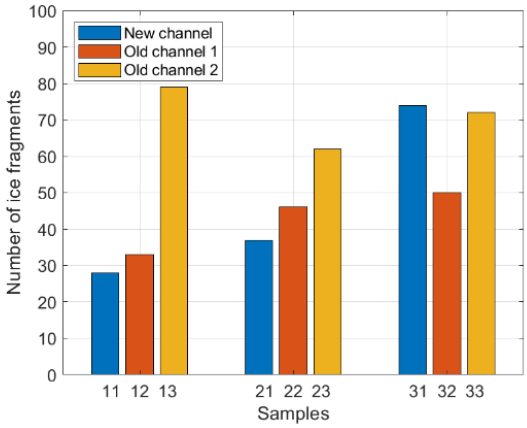
Figure 2: The brash ice sample 11 and 13 displayed on a red grid panel.

Figure 3 presents the ready-use digital binary images for the brash ice shape detection. It shows that the new channel brash ice size is relatively larger than the old channel ice. Some slush appeared in the old channel sample. Figure 4a shows the number of detected ice fragments in the binary images. An ice fragment can be identified if the diameter is over 60 pixels with solidity over 0.4. The diameter is the mean of the major axis length and minor axis length of the ellipse, representing the ice-shaped region. It shows more ice fragments in the old channel than the new channel as some slush is counted. The number of brash ice fragments is used for the following statistical analysis.

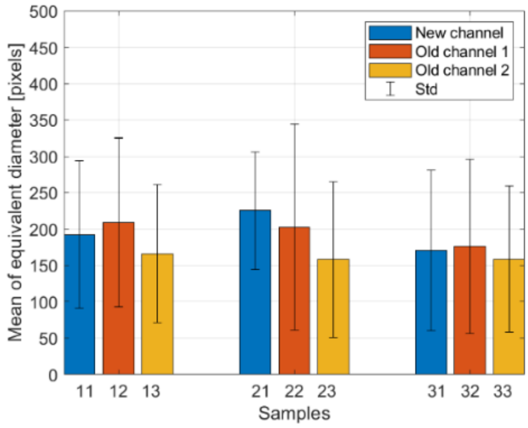


(a) Sample 11 (b) Sample 13

Figure 3: Noise-filtered images of brash ice samples.



(a)



(b)

Figure 4: Brash ice sample detection results: (a) number of ice fragments; (b) mean of equivalent diameter of brash ice.

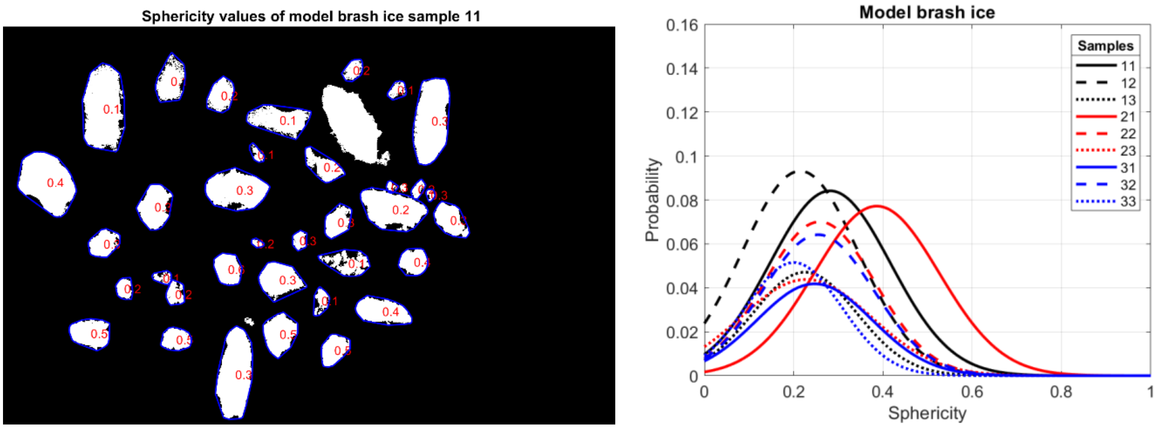
The size of brash ice fragment is also relevant for the shape analysis. The ice fragment size can be described by the equivalent diameter of the circle with the same area as the ice fragment in pixels. Figure 4b illustrates the mean and standard deviation of the diameters of ice fragments in all samples. The mean diameter of brash ice in Old channel 2 is the smallest in all three comparative cases, which corresponds well with the visual observation in model tests: the soft, scaled-down model ice fragments are easily broken after the model ship passing through the channel. Figure 4b also shows that the brash size deviation is quite similar regardless of the ice strength and brash ice types.

The brash ice shape here is characterized by sphericity and roundness. Sphericity and roundness are indices as dimensionless numbers. Many computational approaches have been developed to calculate object sphericity and roundness (Wadell, 1932; Cassel et al., 2018; Zheng and Hryciw, 2015). This paper presents a simple case study on calculating the sphericity and roundness of brash ice in two dimensions. Sphericity of ice fragment is calculated by $c = 4\pi S/perimeter^2$, where S is the object's area. The sphericity of ice fragments can be directly obtained using "regionprops" function in Matlab (circularity).

Figure 5a illustrates the sphericity values of the detected model brash ice of sample 11. Note that a circle's sphericity is 1 and the measured sphericity varies from 0 to 1. Figure 5b shows the sphericity probability distribution of brash ice fragments of nine samples. The line color gathers the samples from the brash ice channels made from the same level ice sheet. The sample number refers to Table 1. In general, the studied brash ice samples appear less circular which is in line with the observation. The mode of sphericity of brash ice sample 11~13 decreases from 0.29 to 0.22, indicating that the brash ice fragments become less circular due to fragment interaction (e.g., crushing and/or friction) by the ship's passage. This trend is similar to the full-scale observation, where the ice fragments become less angular after several ship passages (Sandkvist, 1986). All Old channel 2 (samples 13, 23, 33) seem to have a low probability due to more slush-like brash ice fragments counted (refer to Figure 4a).

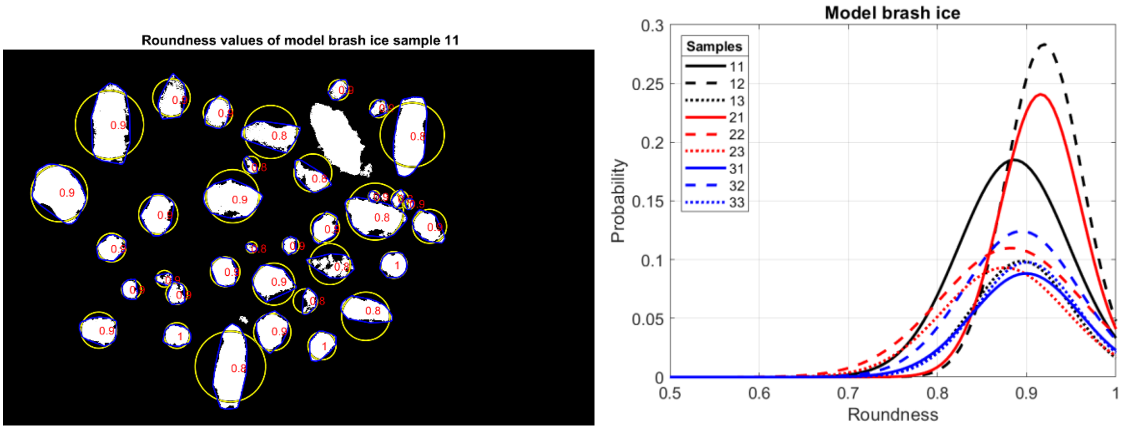
Roundness is calculated by the ray-casting method (Cruz-Matías et al., 2019, Rico et al., 2010). This method considers a least-squares circle (LSC) as the "best circle fits the object contouring points". Roundness is defined as one minus the ratio of the roundness error to the LSC radius (Cruz-Matías et al., 2019). Assuming a series of rays are launching from the object's center to cover the whole object, one ray has intersection points on both the LSC and the object contour. Roundness error is the mean distance between the intersection points of all rays passing through the LSC and the object contour (convex hull), where the distance is positive if the ray intersection point on the object contour is further away from the center than the ray intersection point on the LSC.

Figure 6a illustrates the roundness values of the detected model brash ice of sample 11. Note that a circle's roundness is 1 and the measured roundness varies from 0 to 1. Figure 6b shows the roundness probability distribution of nine brash ice samples. The yellow circle is LSC, and the blue lines contour the convex hull of the brash ice. The roundness of all cases appears larger than 0.6, indicating that the roundness error is less than 0.4 radius of LSC. The mode of the roundness of all samples varies in the range of 0.88 to 0.92. It seems that the roundness of three new channel brash ice samples relates to the level ice flexural strength σ_f. The large σ_f level ice (strong ice) could produce relatively less sharp brash ice fragments than the small σ_f level ice (soft ice). However, the roundness of old channel soft brash ice stands in line with strong brash ice, indicating that the soft ice sharp corners can be easily milled away by the brash ice fragment interaction in the ship performance test. It is also interesting to observe that the roundness of strong brash ice becomes smaller as ship repeating passages increases. More sample analysis is required for further study.



(a) (b)

Figure 5. (a) Sphericity of the ice fragments of sample 11; (b) Sphericity probability distribution of nine samples.



(a) (b)

Figure 6. (a) Roundness of the ice fragments of sample 11; (b) Probability of roundness of nine samples.

5. DISCUSSION

The previous section presents a case study of model brash ice shape characterization and the brash ice shape affected by the repeated ship model testing. This section discusses how to use the brash ice shape characters – roundness as an example – to interpret the change in model brash ice material properties. Note that the roundness is preferred over sphericity since the roundness seems to be a dominant factor in particle-interaction determining material properties.

The variation of calculated mean roundness R_{mean} of model brash ice in three samples collected from three channels (Figure 6b) is first compared among three ice sheets (Table 1). Table 2 lists the R_{mean} of nine samples from Figure 6b. The variation of mean roundness R_{mean} is calculated by $(R_{max} - R_{min}) / R_{max}$, where R_{min} and R_{max} are the minimum and maximum of R_{mean} of ice samples from one ice sheet. It shows that the variation of R_{mean} of different channels in one ice sheet is noticeable but small. For reference, Table 2 also lists a column of roundness varying from angular (R = 0.1) to round (R=1).

Based on Section 3, the following material properties are discussed here: porosity, internal friction angle, and angle of repose. Assuming the relationship between the roundness and material properties observed in granular soil could be applicable in model brash ice, Table 2 lists the evaluation of brash ice roundness influence on the material properties using the empirical formulas in Section 3. The calculated influence value [%] for each property is determined by calculating the property value using the empirical formulas at each sample R values and determining the property value difference between each ice field maximum and minimum value.

This data must be interpreted with caution for two protentional reasons. First, it should be noted whether directly using the empirical formulas of granular soils is appropriate for model-scale brash ice. Second, the mathematical expression of roundness used in this paper is not the same as that in the empirical formulas. However, the "roundness" definition is the same, describing the fragment angularity and the surface roughness.

Table 2: Brash ice sample mean roundness and influence on brash ice properties.

	Ice sheet 1 (sample 11 - 13)	Ice sheet 2 (sample 21 - 23)	Ice sheet 3 (sample 31 - 33)	Roundness reference: R = 0.1…1
New channel R_{mean}	0.886	0.916	0.899	-
Old channel 1 R_{mean}	0.919	0.882	0.894	-
Old channel 2 R_{mean}	0.895	0.875	0.900	-
Variation of R_{mean}	3.66 %	4.44 %	0.68 %	90.00 %
Calculated influence on porosity	1.50 %	1.80 %	0.27 %	25.62 %
Calculated influence on internal friction angle	2.12 %	2.55 %	0.39 %	37.97 %
Calculated influence on angle of repose	1.67 %	2.01 %	0.31 %	32.19 %

Table 2 shows that the roundness has an insignificant influence on the studied brash ice material properties. In other words, the model brash ice shape changed by repeated ship model testing would not significantly affect the brash ice properties in model scale. By comparison, the reference values of fragment shape roundness varying from very angular (R = 0.1) to perfect round (R = 1) would substantially affect the studied material properties. Assuming the brash ice in full scale behaves similarly to the granular soil, the effect of roundness on the material properties would be pronounced as the reference values indicated, such as porosity changed by 26 %. If model-scale brash ice could mimic the brash ice in nature, such an effect on material properties could also be observed. This analysis encourages us to consider the ice fragment shape characters as target geometrical properties for modeling brash ice for ship model testing. Defining the target roundness range of the model brash ice requires further studies, which could contribute to standardize the ship model testing procedures in brash ice channels.

5. CONCLUSIONS

This paper studies the effect of the model-scale brash ice shape on the material properties for ship model testing by reviewing and applying the shape effect knowledge of the granular soil. The paper presents a preliminary approach to characterize model brash ice fragment shape – sphericity and roundness – from digital images. This study reveals that the repeated ship model testing changes the brash ice sphericity and roundness measured by computational geometric methods. This paper also attempts to discuss the effect of ice fragment roundness on the material properties by using the empirical formulas of granular soil when assuming the model brash ice behave similarly to the granular soil. The noticeable variation of roundness of model brash ice in different channels has an insignificant influence on the studied material properties. This paper suggests considering the ice fragment shape characters as target geometrical properties for standardizing ship model testing in brash ice channels.

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