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STUDY AND MODELLING OF BEHAVIOR AND SPREADING OF OIL IN COLD WATER AND IN ICE CONDITIONS

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ABSTRACT

This paper shows the results of mathematical modelling connected with oil spreading in ice and cold water conditions. Brief description of the physical basis, mathematical formalization and original numerical technique for oil spreading model are presented. The governing equations for oil spreading at the water surface are supplemented by the additional terms describing the viscous stresses in oil slick. The boundary of oiled area is considered to be unknown, and is determined in the process of solution. The particles-in-cell technique on quasi-Eulerian adaptive grids is used. The problems of the model developing and tuning are discussed. The results of modelling are discussed to estimate a validity of derived equations in oil behavior description.

1. INTRODUCTION

The study of oil spill behavior in cold water and ice conditions is made as a logical continuation of the oil spill spreading investigations carried out over the last few years.

Oil spreading in cold water differs from traditional approach because of considerable viscous-temperature dependence and increasing of the role of internal viscous stresses. In spite of this fact there are no known practical models with oil viscosity used as a parameter for calculations. Cox&Di Pietro (1980) have begun the mathematical investigation of this problem. Venkatesh et al. (1992) has attempted to integrate data of full-scale and laboratory experiments in research of oil spreading and has offered relationship between final equilibrium thickness of oil spill and oil viscosity. This paper is the attempt to combine under general mathematical roof the different mathematical problems arising in oil spill simulation.

2. MODEL DESCRIPTION

Oil spill is represented in the model as sufficiently thick lens and very thin layer (often called as monolayer) around it. The most part of oil (about 90 %) is contained in the lenses. In most practical applications the transport of lenses and their shape and size evolution are more important than evolution of thin monolayer. Model described below includes the basic transformation processes for oil lens dynamics. Theoretical analysis of spreading process is made for *calm sea state condition*. The oil inside the lens is represented as a *homogeneous Newtonian liquid.*

The first assumption in the mathematical formulation of the spreading process is (***I***) that thickness H of spreading liquid is much less than horizontal process scale B, and their ratio

H/B<<1 may be chosen as an expansion procedure parameter to simplify the original Navie-Stokes equations. The next simplification, which follows from the first one, is (***II***) that surface tension effects are negligible because of the small surface curvature. Also we assume that (***III***) interaction between bulk layer (lenses) and monolayer *is supposed negligible.*

Assumptions (I)- (III) allow to obtain governing equations for oil spreading under different external conditions. Cox&Di Pietro (1980) submitted the complete expansion procedure for viscous oil spreading in detail. The viscous stresses was represented by the term

$$L(H,\mathbf{u}) = \mu(\Delta\mathbf{u} + \frac{1}{H}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T).\nabla H + \frac{2}{H}(\nabla.\mathbf{u}\nabla H) + 3\nabla\nabla.\mathbf{u})$$

where $\mathbf{u}$ is oil velocity vector , H is oil thickness, μ is oil viscosity, $\nabla.$ is horizontal divergence operator, ∇ is horizontal gradient operator,

By means of some formal mathematical procedure this term can be rewritten as a two-dimensional stress tensor with components

$$T_{xx} = \mu H(4\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y}) \qquad T_{xy} = \mu H(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$$

$$T_{yx} = \mu H(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \qquad T_{xx} = \mu H(2\frac{\partial u}{\partial x} + 4\frac{\partial v}{\partial y})$$

In case when the role of internal viscous stresses is considerable, the governing equation for oil spreading [Zatsepa,et al.,1992] (momentum balance) may be written as

$$\rho_o H(\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\nabla\mathbf{u}) = -\rho_o\lambda gH\nabla H - \beta|\mathbf{u}|\mathbf{u} + \nabla.\mathbf{T}$$

where ρ_o is oil density , β is empirical coefficient , $\lambda = \frac{\rho_o - \rho_w}{\rho_w}$, g is gravitational constant, ρ_w is the water density, $\mathbf{T}$ is two dimensional viscous stress tensor.

When the viscous term $L(H,\mathbf{u})$ is written in tensor form it is easy to formulate conditions at free boundary, which imply that the tangential and normal component of the stress are continuous and equal to zero at the free boundary.

If we denote ρH as ρ' - "density" of the new two-dimensional media and denote μH as μ' (new viscosity) , it is convenient to rewrite the stress tensor as isotropic part and deviator **S**

$$\mathbf{T} = (\frac{g}{\rho}\rho'^2 + 3\mu'\nabla.\mathbf{u})\mathbf{I} + \mu'\mathbf{S}$$

where **S**

$$S_{xx} = 2\frac{\partial u}{\partial x} - \nabla.\mathbf{u} \qquad S_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$S_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad S_{yy} = 2\frac{\partial v}{\partial y} - \nabla.\mathbf{u}$$

Thus the system of governing equations for oil spreading in two-dimensional area $\Omega(t,\mathbf{x})$, $\mathbf{x}=(x,y)$ are as follows :

$$\rho_o H(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\nabla\mathbf{u}) = \nabla(-\rho_o \lambda g H^2 + 4\mu H \nabla.\mathbf{u}) - \beta|\mathbf{u}|\mathbf{u} + \nabla.\mathbf{S} \tag{1}$$

Ω:

$$\frac{\partial H}{\partial t} + \nabla.(H\mathbf{u}) = \frac{(k_e + k_w - Q)}{\rho}, \tag{2}$$

where k_e is mass flux due to evaporation, k_w is mass flux due to breaking waves, Q is oil flux from spill source. Oil evaporation model in case of high oil viscosity should be changed because of small diffusion coefficient.

If the boundary of $\Omega(t,x,y)$ is $L(t,x,y)$ and $L=L_1(t,x,y) \cup L_2(x,y)$, where L_1 is a free boundary and L_2 is contact (solid) boundary, it is necessary to use the following boundary conditions:

$$L1: \quad R_t + \mathbf{u}\nabla R - \frac{(k_e + k_w)}{\rho}\frac{|\nabla H||\nabla R|}{(\nabla H \nabla R)}\nabla R = 0 \tag{3}$$

(kinematics condition),

where $R(x,y,t) = 0$ is the equation of free boundary and

$$L_1: \qquad H\big|_{x,y\in R} = 0 \tag{4}$$

(dynamic condition)

The first two terms of equation (3) describe the movement of free boundary due to movement of liquid particles inhering the boundary. The last term describes the movement of free boundary due to evaporation and vertical dispersion processes.

At fixed (contact) boundary L_2 (if any) must be

$$L_2: \qquad \mathbf{u}\big|_{x,y\in L_2} = 0 \ \text{ if } H>0 \tag{5}$$

The system (1)-(5) must be combined with the following initial conditions:

$$\Omega(x,y,t) = \Omega_0(x,y)$$

$$H(x,y,t) = H_0(x,y),\ \mathbf{u}(x,y,0) = 0 \quad x,y \in \Omega_0 \tag{6}$$

where Ω_0 - area covered by oil at $t = 0$.

It is necessary to determine $H, \mathbf{u}, \Omega$ for $t > 0$.

The system (1)-(6) is open because of undetermined mass fluxes due to evaporation and dispersion processes. For simplicity of further discussion this mass fluxes were assumed to equal zero.

3. NUMERICAL EXPERIMENTS

For evaluation of a role of viscosity on spreading processes in the frame of equations (1)-(2) with boundary and initial conditions (3)-(6), mentioned above, two groups of numerical experiments were carried out. In the first group of experiments of spreading of oil with various viscosity (from 0 up to 5 m^2/s) without interaction at oil-water boundary have been carried out. Thus at this series of experiments gravity forces are balanced by internal viscous stresses and inertia. All numerical experiments had started from identical initial condition. Initial thickness axisymmetrical distribution of oil in all experiments was set as

$$H(r,0) = H_0(1 - (\frac{r}{r_0})^2) \tag{7}$$

where $H_0 = 0.28\,m \quad r_0 = 10\,m$.

All calculations were carried out in Cartesian co-ordinate system. The axisymmetrical initial distribution of thickness used, as most simple for matching with the idealised self-similar solutions, if those exist. The Eulerian-Lagrangian numerical scheme used here is based on particle-in-cells technology (PIC), but differs from the original one developed by Harlow (1964). The details of scheme were discussed in (Ovsienko et al., 1995). The results of experiments are submitted on Fig. 1.

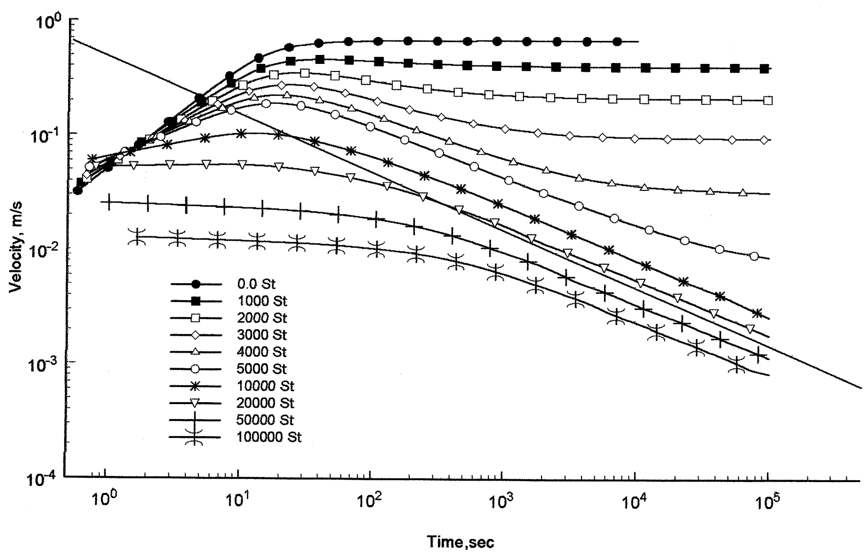

Figure1 Spreading velocity vs. time for "no external drag" spreading

At the above figure for each of curves it is possible to dedicate three areas: 1) initial acceleration segment from a state of rest 2) intermediate area, where main operational forces are viscosity and pressure gradient; 3) inertial area. The straight line on the plot is

$$u(t) \propto t^{-\frac{1}{2}} \tag{8}$$

and it is shown for evaluation of the slope of experimental curves at the intermediate stage of spreading. The simple explanation of this fact can be as follows. In a radially symmetrical case of a spreading the balance of gravity force and viscous stresses can be written as

$$\rho_o g' \nabla H = \mu \left[4\frac{\partial^2 u}{\partial r^2} + 4\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\nabla H}{H}\left(\frac{u}{r} + 2\frac{\partial u}{\partial r}\right) - 3\frac{u}{r^2} \right] \tag{9}$$

Let's remark, that all terms of a right side of an equation have an order of magnitude

$$\mu \left[4\frac{\partial^2 u}{\partial r^2} + 4\frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{H}\frac{\partial H}{\partial r}\left(\frac{u}{r} + 2\frac{\partial u}{\partial r}\right) - 3\frac{u}{r^2} \right] \propto \frac{u}{r^2} \propto \frac{1}{rt} \tag{10}$$

and left side of an equation have an order of magnitude

$$\rho_o g' \frac{\partial H}{\partial r} \propto \frac{1}{r^3}, \text{ because oil volume } \Omega_{oil} \propto H r^2 \tag{11}$$

and therefore

$$\frac{1}{r^3} \propto \frac{u}{r^2} \propto \frac{r}{t}\frac{1}{r^2} \Rightarrow r^2 \propto t \Rightarrow u \propto t^{-\frac{1}{2}} \tag{12}$$

The upper curve on the graph corresponds to case $\nu = 0$ and describes free expansion of oil with edge velocity $u = \sqrt{g' H_0}$. It is seen from a plot, that after the first stage of oil spreading, when under oil pressure gradients action in a layer of oil the velocity distribution was formed, the "internal" friction forces result in deboosting a spreading. At the last stage the spreading velocity aims to a constant. A reason of such behaviour of spreading velocity can be fast decreasing (practically up to zero point) viscous stresses, both owing to decreasing gradients of flow velocities, and to reduction viscosity of modelled media, defined above. The time of approach of a inertial interval for each of curves is various and depends on value of a coefficient of viscosity.

The essentially other picture is observed with oil -water friction presence. (Fig.2).

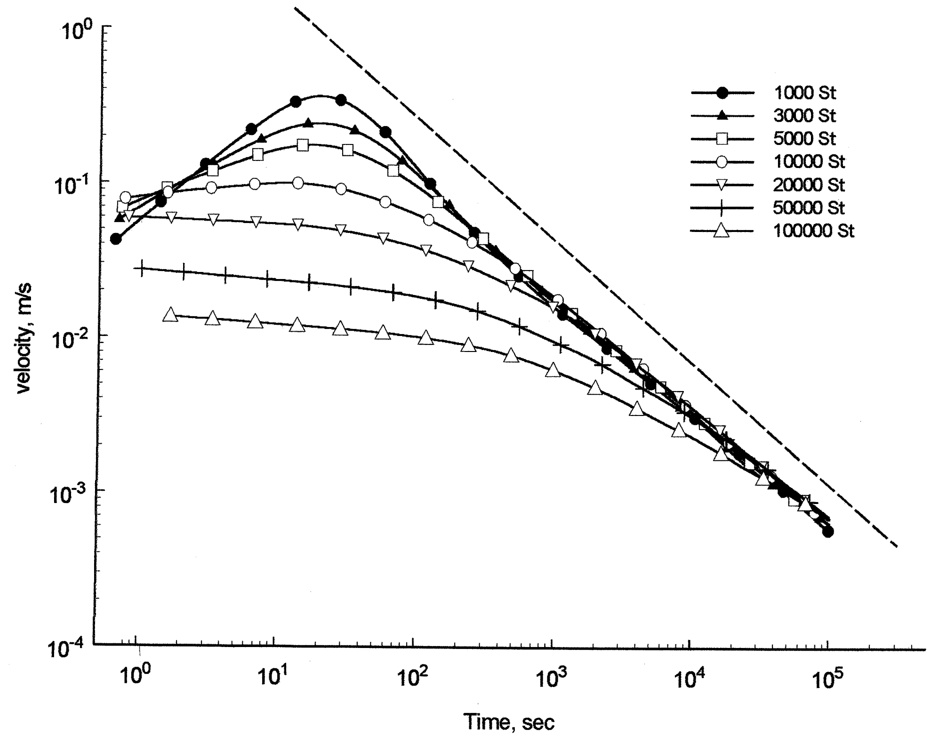

Figure 2. Spreading velocity vs. time in presence oil-water friction

The asymptotic mode for all curves is

$$u_{spread} \propto t^{-\frac{5}{7}} \tag{13}$$

which corresponds with simple evaluating procedure

$$Hg\frac{\partial H}{\partial r} = \beta u^2 \Rightarrow \left\{H \propto \frac{\Omega}{R^2}\right\} \Rightarrow g\frac{\Omega}{R^2}\frac{\Omega}{R^3} \approx \beta\frac{R^2}{t^2} \Rightarrow R^7 \propto t^2 \Rightarrow R \propto t^{\frac{2}{7}} \Rightarrow u \propto t^{-\frac{5}{7}}$$

Friction on oil-water boundary becomes the dominant resistance force, because the effect of friction force on oil-water boundary is inversely proportional to average oil thickness.

The slick sizes depending on viscosity are distinguished rather considerably. We shall emphasise, that all submitted evaluations are corresponded to instantaneous spill of 3.14 m^3 of oil with initial radius of 10 meters.

The submitted above problem statement permits to simulate dynamics of oil with various properties in the domains with complex geometry. Spreading of oil in brush ice considered as movement of oil in spaces of open water between floes. This process can be simulated directly, in the frameworks of above-stated mathematical problem, however, it requires significant computing resources. Direct modelling of spreading process between floes is interesting for spreading process research for subsequent averaged equations derivation of oil in ice movement, in which such macro-characteristics of ice, as compactness, size of floes will appear. The example of similar experiment is presented on Fig. 4.

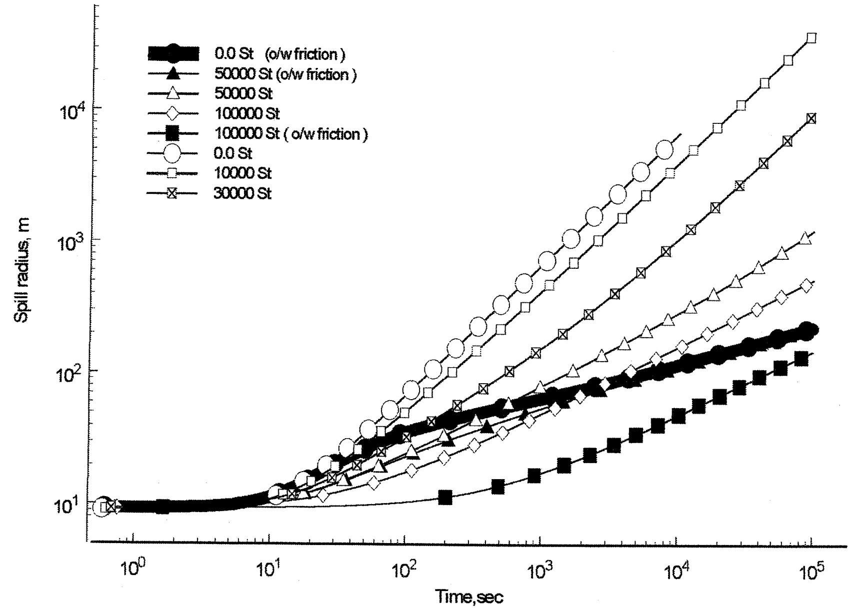

Figure 3. Spill radius vs. time for different cases
- light markers correspond "no external drag" case
- dark markers correspond cases with oil-water boundary friction

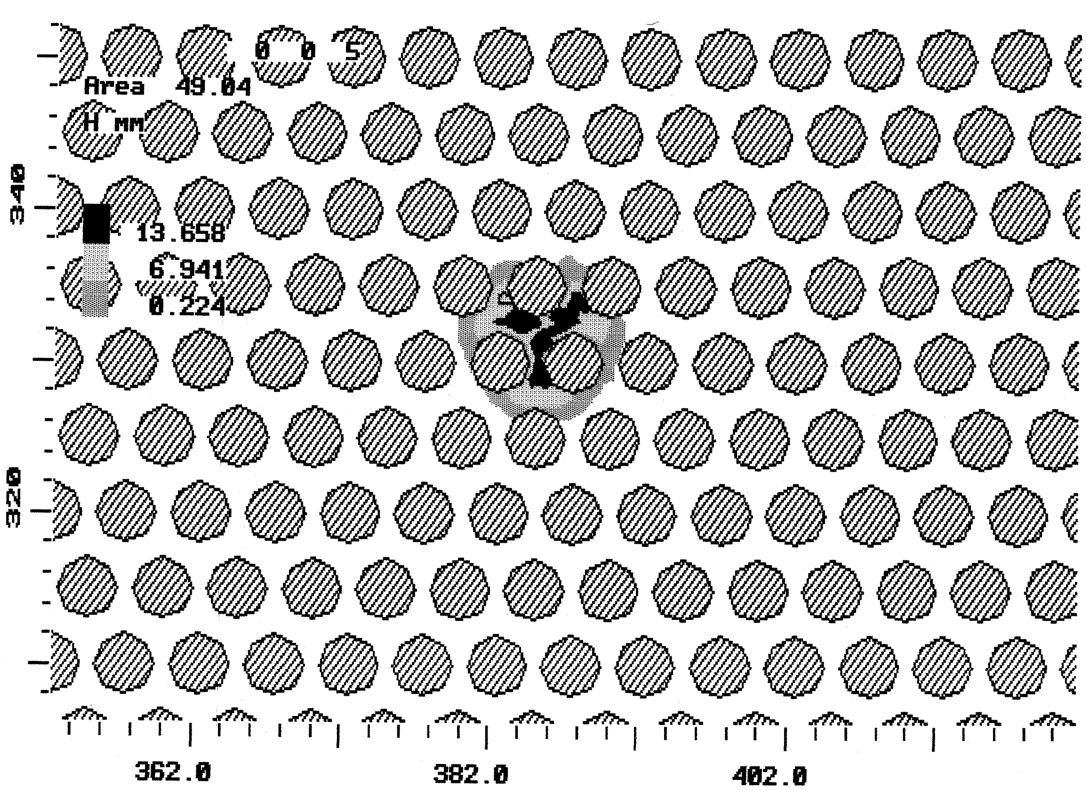

Figure 4. Pattern of oil spreading between "floes"

An ice field is simulated by a set of disks, located in hexagonal grid nodes arranged at the distance of 5 meters from each other. Compactness of such field can is $\frac{\pi\rho^2}{\sqrt{3}/2}$, where ρ is the ratio of disk's diameter to distance between disks. Maximum compactness of such field (ρ=0.5) is 0.9069. In all experiments spill of 1 m^3 of oil was simulated in spaces between floes. Initial radius of oil spill was 5 meters. The configuration of oil field after t=100 s is shown on Fig.5.

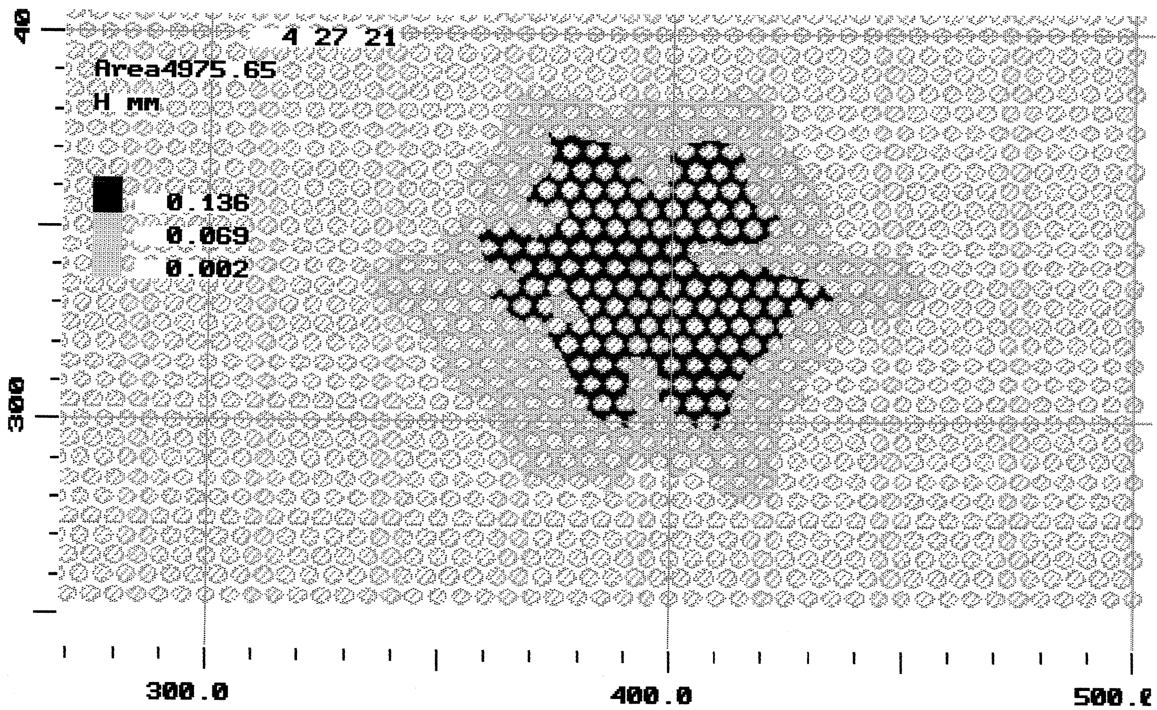

Figure 5 Pattern of oil spreading between ice floes at t=10^2

The submitted pictures only illustrate the scope of the model realisation for oil behaviour investigation in ice pack conditions..

4. CONCLUSIONS

The mathematical model of a spreading of oil formulated as a problem of a spreading dynamics of a limited volume of a "two-dimensional" liquid with free boundaries, was suggested for investigation of oil spreading dynamics in relation to external conditions and internal parameters. In cases, when the simulation results mismatch with observed behavior of oil it is necessary to pay attention to a feasibility of simplifying assumptions, used for mathematical reduction of a problem.

It is found, that the model can describe existing analytical solutions. In addition this model can also describe sufficiently more complicated situations, first of all it can calculate spills in regions with complicated geometry. It allows also to take into account various processes, such as, for example, the oil properties change at low temperature during the spreading process. It seems prospective to use the model for planning the field and laboratory experiments and for evaluation of their results. The model is compatible with oil spreading model for calm water. It seems reasonable to use the similar model for describing the oil spill under ice, in broken ice and on the ground.

5. REFERENCES

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