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AN ANALYSIS OF CRACK NUCLEATION DURING CREEP OF S2 COLUMNAR ICE UNDER UNIAXIAL COMPRESSION

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ABSTRACT

A model is proposed to assess the influence of the viscoplastic anisotropy of ice on the formation of cracks in S2 columnar ice under uniaxial compressive steady creep. Each grain is considered as an anisotropic inclusion embedded in a homogeneous medium exhibiting the mean properties of the polycrystal. By assuming a linear behaviour of ice (*i.e.* a linear relation between the deviatoric and strain-rate tensors), analytical expressions are obtained for the maximum principal stresses inside a grain with a given crystallographic orientation, and in the matrix. It is shown that, under uniaxial compression, an easy-glide oriented grain can induce a tensile stress in the matrix, at the inclusion-matrix interface, which is high enough to allow the formation of a crack parallel to the direction of loading, while the shear stress acting on the basal plane of such a grain seems too low to induce transcrystalline cracking.

1. INTRODUCTION

Several mechanisms and models have been proposed to explain the observations of crack nucleation in S2 columnar ice under compressive stress in the brittle and ductile regimes. Sinha (1984) proposed a theory linking crack nucleation to a critical delayed elastic strain associated with grain boundary sliding. Nickolayev and Schulson (1995) observed grain boundary sliding in S2 ice with columns inclined at 45° with respect to the direction of compression, leading to across-column cracking. Picu and Gupta (1995) observations on columnar ice loaded perpendicular to the long direction of the grains, favour the nucleation of wing cracks at triple junctions owing to grain boundary sliding. Other models for crack nucleation in the brittle regime are based on the elastic anisotropy of the ice grains (Cole, 1988; Shyam Sunder and Wu, 1990; Wu and Niu, 1995). According to these authors, the mismatch in the deformation of two adjacent grains with different crystallographic orientations, which results from the elastic anisotropy, gives rise to a singularity of the local stress. Shyam Sunder and Wu (1990) gave an estimate of the microstructural stress field acting at the grain scale by using Eshelby's (1957) analysis, and expressed the problem of crack nucleation in terms of the growth of a pre-existing precursor to a crack, propagation being allowed when a maximum principal stress criterion is satisfied.

In the ductile range, the ice single crystal deforms essentially by dislocation glide on the basal plane, normal to the hexagonal symmetry c-axis, and consequently exhibits a very strong viscoplastic anisotropy. According to Duval et al. (1983), non-basal deformation requires stresses at least 60 times larger than those for an easy-glide oriented crystal under the same strain-rate at −10 °C. By testing columnar ice in the ductile regime, Gold (1972) observed that cracks were formed for creep compressive stresses above 0.6 MPa. Gold's (1972) analysis underlined the existence of two populations of cracks: one family consists of cracks formed at grain boundaries upon loading of the specimens, and can be explained by the mechanisms responsible for the nucleation of cracks in the brittle regime; the second family consists of transcrystalline cracks which result from creep-dependent mechanisms. The formation of transcrystalline cracks in the ductile regime is generally explained by the dislocation pile-up mechanism (Gold 1972; Cole 1988; Kalifa et al. 1989), and criteria for crack initiation are obtained by assuming that a dislocation pile-up acts as a crack which induces a singularity of the stress field at its head, sufficient enough to initiate a crack in the adjacent grain (*e.g.* Stroh, 1957).

The present study aims to assess the ability of the viscoplastic anisotropy of ice alone, that is without invoking the singularity of the stress field arising at the end of a pile-up, to be responsible for stress concentrations high enough to initiate cracks in the ductile regime. To do so, following Shyam Sunder and Wu (1990), the microstructural stress field is obtained by using the Eshelby's model of inclusion, and a maximum principal stress criterion for crack propagation. In the first stage presented here, the analysis is restricted to linear behaviour and steady state. Although the creep rate of ice follows a power law with exponent n=3 for stresses greater than 0.1 MPa (Duval et al., 1983), the restriction to n=1 allows a complete analytical development which can help in the understanding of the problem, while giving a correct order of magnitude for the stress concentrations arising from viscoplastic anisotropy (see section 5). The present analysis is done for steady creep and focuses on the nucleation of the second family of cracks as defined by Gold (1972) (*i.e.* the mechanisms arising upon loading or during transient creep, such as grain boundary sliding, are not considered).

2. THE INCLUSION MODEL

To obtain an estimate of the stress field existing at the grain scale during steady creep, each grain of the S2 ice polycrystal is considered as an inclusion in a homogeneous medium exhibiting the mean properties of the polycrystal (see Figure 1).

Owing to the anisotropy of the single crystal, the viscoplastic deformation of a grain loaded in a plane perpendicular to its basal planes can be considered as two dimensional. Since the *c*-axis of each grain is perpendicular to its long axis, the plane strain assumption can be extended to model the flow of S2 ice loaded in the plane perpendicular to the long direction of the grains (Gold, 1997, Plé and Meyssonnier, 1997). As a consequence, the inclusion problem can be treated as that of a two-dimensional circular anisotropic inclusion embedded in a two-dimensional equivalent medium.

2.1. Grain and matrix behaviour

Each grain is modelled as a transversely isotropic incompressible medium, whose symmetry axis is the *c*-axis of the crystal. In the linear case this can be shown to be exactly equivalent to the classical approach used in polycrystal plasticity which involves the basal, prismatic and pyramidal slip systems (Meyssonnier and Philip, 1999a). In the non linear case, this simplification of the actual behaviour of the ice crystal is justified by Kamb (1961). Since the *c*-axis of each grain is oriented randomly in the plane perpendicular to its long direction, S2 ice is also transversely isotropic, with its symmetry axis parallel to the long direction of the grains.

The general constitutive relation linking the deviatoric stress tensor $\boldsymbol{s}$ and the strain-rate tensor $\boldsymbol{D}$, for a transversely isotropic incompressible linear medium with symmetry axis along the x_3 direction, is written as (see for instance Lliboutry, 1993; Meyssonnier and Philip, 1999b)

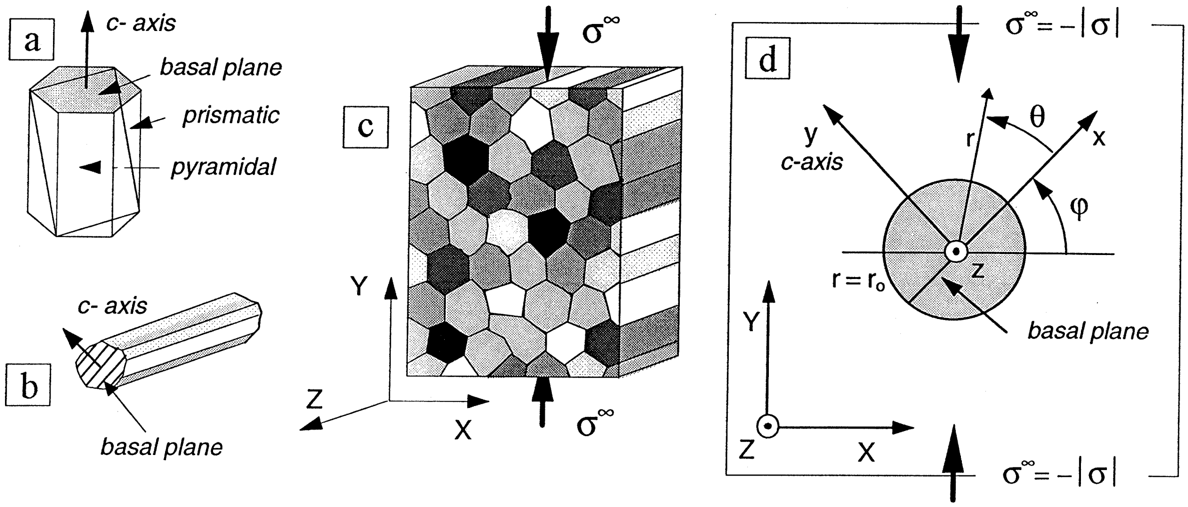

Figure 1: a) ice single crystal crystallographic planes ; b) columnar grain of S2 ice (the *c*-axis is perpendicular to its long direction) ; c) S2 ice specimen loaded in the plane perpendicular to the long direction of the columns ; d) the inclusion model : the fixed global reference frame *{X,Y,Z}* is attached to the S2 polycrystal ; *{x,y,z}* is attached to the circular inclusion $(r = r_0)$ of orientation φ ; $\{r,\theta\}$ are polar co-ordinates in the local reference frame *{x,y,z}*.

$$s_{11} = 2\eta_{12}\,[D_{11} - \frac{2}{3}(\alpha-1)D_{33}]\,,\; s_{22} = 2\eta_{12}\,[D_{22} - \frac{2}{3}(\alpha-1)D_{33}]\,,\; s_{12} = 2\eta_{12}\,D_{12}$$

$$s_{33} = 2\eta_{12}\,[1 + \frac{4}{3}(\alpha-1)]\,D_{33}\,,\quad s_{23} = 2\eta_{12}\,\beta D_{23}\,,\quad s_{31} = 2\eta_{12}\,\beta D_{31}, \qquad (1)$$

where η_{12} is the viscosity for shear in the plane of isotropy, α is the ratio of the axial viscosity along the x_3-axis of symmetry to the axial viscosity in the plane of isotropy (x_1,x_2) (*i.e.* viscosities corresponding to uniaxial compression along these axes), and β is the ratio of the viscosity for shear parallel to the plane (x_1,x_2) to the viscosity for shear in the plane (x_1,x_2).

Relation (1) models the grain (inclusion) behaviour, the x_3-axis corresponding to the grain *c*-axis, as well as the S2 polycrystal (matrix) behaviour, the x_3-axis corresponding then to the long direction of the columns.

2.2. Solution of the inclusion problem

Two reference frames are used to solve the plane strain problem: a local reference frame *(x,y,z)* attached to the grain, a fixed global reference frame *(X,Y,Z)* attached to the homogeneous matrix which represents the polycrystal of S2 ice. The long direction of the columns is along the *z*-axis of the local reference frame, which coincides with the *Z*-axis of the global reference frame. The *c*-axis of the inclusion coincides with the *y*-axis, and the trace of the basal plane is along the *x*-axis, at angle φ from the *X*-axis of the global reference frame (see Figure 1). We consider only loading in the $(X,Y)=(x,y)$ plane.

In the following, the viscosity is noted η_i in the basal plane of the inclusion, and η in the *(X,Y)* plane of isotropy of the matrix. Both correspond to η_{12} in relation(1), in which the indexes *(1,2,3)* correspond to *(z,x,y)* for the inclusion, and to *(X,Y,Z)* for the matrix.

Following Meyssonnier and Philip (1999b), the solution of the inclusion problem can be easily achieved by using polar co-ordinates (r,θ), the polar angle θ being measured from the x-axis attached to the inclusion (see Figure 1). With the notation $\xi = r/r_0$, where r_0 is the radius of the inclusion, and for strain-rates D^{∞}_{xx}, $D^{\infty}_{yy} = -D^{\infty}_{xx}$ and D^{∞}_{xy} prescribed at infinity and expressed in the local reference frame of the inclusion (and satisfying the plane strain condition, *i.e.* $D^{\infty}_{zz}=D^{\infty}_{xz}=D^{\infty}_{yz}=0$), the solution satisfying the continuity of the velocity and of the stress vector at the interface $\xi = 1$ between inclusion and matrix, is obtained for the points inside the inclusion, and in the local reference frame, as

$$\begin{aligned} &D_{xx} = (1+K_\alpha)D^{\infty}_{xx}, \quad D_{yy} = -D_{xx}, \quad D_{xy} = (1+K_\beta)D^{\infty}_{xy}, \quad D_{zz} = D_{xz} = D_{yz} = 0, \\ &p = (\sigma_1+\sigma_2+\sigma_3)/3 = p^{\infty} + 2\eta_i(\alpha-1)(1+K_\alpha)D^{\infty}_{xx}/3, \end{aligned} \tag{2}$$

where

$$K_\alpha = \frac{1-\lambda\alpha}{1+\lambda\alpha}, \quad K_\beta = \frac{1-\lambda\beta}{1+\lambda\beta}, \quad \lambda = \frac{\eta_i}{\eta}. \tag{3}$$

From relation (2) it can be seen that the strain-rate inside the inclusion is uniform, which is in accordance with Eshelby's (1957) result. Taking into account relation (1) with the x_3-axis corresponding to the y-axis and $D_{22}=0$, the solution for the stresses inside the inclusion derived from (2) follows as

$$\begin{aligned} &s_{xx} = \lambda(1+2\alpha)(1+K_\alpha)s^{\infty}_{xx}/3, \quad s_{yy} = \lambda(1-4\alpha)(1+K_\alpha)s^{\infty}_{xx}/3, \quad s_{zz} = -(s_{xx}+s_{yy}) \\ &s_{xy} = \lambda\beta(1+K_\beta)s^{\infty}_{xy}, \quad s_{xz} = s_{yz} = 0, \quad p = p^{\infty} + \lambda(\alpha-1)(1+K_\alpha)s^{\infty}_{xx}/3. \end{aligned} \tag{4}$$

Taking into account relation (1) with the x_3-axis corresponding to the Z-axis and $D_{33}=0$, the solution for the stresses at "exterior" points situated in the matrix is obtained as (see Meyssonnier and Philip, 1999b for details)

$$\begin{aligned} &s_{rr} = [1+K_\alpha g(\xi)]s^{\infty}_{xx}\cos 2\theta + [1+K_\beta g(\xi)]s^{\infty}_{xy}\sin 2\theta, \\ &s_{\theta\theta} = -s_{rr}, \quad s_{ZZ} = s_{\theta Z} = s_{rZ} = 0, \\ &s_{r\theta} = -[1-K_\alpha g(\xi)]s^{\infty}_{xx}\sin 2\theta + [1-K_\beta g(\xi)]s^{\infty}_{xy}\cos 2\theta, \\ &p = p^{\infty} - 2(K_\alpha s^{\infty}_{xx}\cos 2\theta + K_\beta s^{\infty}_{xy}\sin 2\theta)\xi^{-2}, \\ &g(\xi) = 3\xi^{-4} - 2\xi^{-2}, \quad \xi = r/r0. \end{aligned} \tag{5}$$

2.3. Self-consistency

The mean viscosity η of the polycrystal is obtained by expressing that the mean strain-rate over all the orientations of its constituent grains equals the strain-rate prescribed at infinity on its boundary. Note that in the case of a linear behaviour, expressing the self-consistency on the stress leads to the same result (Meyssonnier and Philip, 1996). Expressing the self-consistency requires to re-write relation (2) for the strain-rate in the inclusion in the global frame, as

$$D_{XX} = [(1+K_\alpha)\cos^2 2\varphi + (1+K_\beta)\sin^2 2\varphi]\, D_{XX}^\infty + (K_\alpha - K_\beta)\sin 2\varphi \cos 2\varphi\, D_{XY}^\infty ,$$

$$D_{YY} = -D_{XX},\quad D_{ZZ} = D_{XZ} = D_{YZ} = 0, \tag{6}$$

$$D_{XY} = (K_\alpha - K_\beta)\sin 2\varphi \cos 2\varphi\, D_{XX}^\infty + [(1+K_\alpha)\sin^2 2\varphi + (1+K_\beta)\cos^2 2\varphi]\, D_{XY}^\infty ,$$

$$p = p^\infty + 2\eta_i(\alpha - 1)(1+K_\alpha)(\cos 2\varphi\, D_{XX}^\infty + \sin 2\varphi\, D_{XY}^\infty)/3 ,$$

The mean value of D being defined by

$$\langle D \rangle = \frac{1}{\pi}\int_0^\pi D(\varphi)d\varphi , \tag{7}$$

the self-consistency conditions on D_{XX} and D_{XY} lead to the same equation

$$K_\alpha + K_\beta = 0 , \tag{8}$$

whose solution is (taking relation (3) into account)

$$\lambda = 1/\sqrt{\alpha\beta} , \tag{9}$$

the self-consistency condition $\langle p \rangle = p^\infty$ being satisfied.

2.4. Grain anisotropy parameters

Following Meyssonnier and Philip (1996), we make the simplifying assumption $\alpha=1$. This means that the difference in the behaviour of the single crystal loaded in compression along the c-axis or in the (isotropic) basal plane, is not considered as crucial. This is supported by the creep data for nonbasal glide in single crystals reported by Duval et al. (1983). With $\alpha=1$, relation (1) reduces to the simplest form of anisotropic behaviour characterised only by β which becomes the ratio of the viscosity for shear parallel to the basal plane to the viscosity in the other directions. The essential feature of the grain anisotropy, that is a weak resistance to shear parallel to the basal plane, is preserved as far as β is much less than 1.

According to Pimienta et al.(1987), granular ice with c-axes aligned along the vertical deforms ten times faster than isotropic ice, when sheared parallel to the basal planes of the grains. In the framework of the self-consistent method, a textured polycrystal with all its all c-axes aligned along the same direction is equivalent to a single grain, then this experimental result can be interpreted in terms of the ratio of the viscosity for shear parallel to the basal plane of the grain $\beta\eta_i$, to the viscosity η of isotropic ice, that is $\beta\eta_i/\eta = \lambda\beta = 0.1$. Since the viscosity of S2 ice in its plane of isotropy is of the same order of magnitude than that of isotropic granular ice (*e.g.* Duval and al., 1983; Sinha, 1984), we adopt the value $\lambda\beta = 0.1$ for describing the grain anisotropy. To satisfy the self-consistency equation (9), the grain anisotropy parameter β corresponding to $\alpha = 1$ and $\lambda\beta = 0.1$, must be $\beta = 0.01$.

3. UNIAXIAL COMPRESSION

In the following we adopt the notation derived from equation (8) :

$$K = K_\alpha = -K_\beta \tag{10}$$

Under uniaxial compression $\sigma^{\infty}=\sigma$ ($\sigma<0$) along the Y-axis, the pressure and deviatoric stresses are given by

$$p=\frac{\sigma}{2}\,,\quad s_{XX}^{\infty}=-\frac{\sigma}{2}\,,\quad s_{YY}^{\infty}=\frac{\sigma}{2}\,,\quad s_{ZZ}^{\infty}=s_{XY}^{\infty}=s_{YZ}^{\infty}=s_{XZ}^{\infty}=0\,, \tag{11}$$

and in the local reference frame of the inclusion:

$$s_{xx}^{\infty}=-\frac{\sigma}{2}\cos 2\varphi\,,\quad s_{xy}^{\infty}=\frac{\sigma}{2}\sin 2\varphi\quad . \tag{12}$$

3.1. Principal stresses in the inclusion

With $\alpha=1$, and using notation (10) and relations (11) and (12), the stresses in the inclusion, expressed in the local reference frame attached to the inclusion by relation (4), become

$$s_{xx}=-\lambda\,(1+K)\frac{\sigma}{2}\cos 2\varphi\,,\quad s_{xy}=\lambda\,(1-K)\frac{\sigma}{2}\sin 2\varphi\,,\quad s_{yy}=-s_{xx}\,,\quad s_{zz}=0,\; p=\frac{\sigma}{2} \tag{13}$$

The corresponding principal stresses are

$$\begin{aligned}&\sigma_1=p+\Delta\,,\quad \sigma_2=p-\Delta\,,\quad \sigma_{zz}=p\,,\\ &\Delta=\sqrt{s_{xx}^2+s_{xy}^2}=\lambda\,\sqrt{(1+K)^2\cos^2 2\varphi+\beta^2(1-K)^2\sin^2 2\varphi}\;\left|\sigma\right|/2\end{aligned} \tag{14}$$

Figure 2a shows the principal stresses in the inclusion as a function of its crystallographic orientation. The maximum principal stress is maximum for $\varphi=0$ and $\varphi=90°$, *i.e.* for grains which are not well oriented for basal glide, taking the value

$$\sigma_1=(\lambda\,(1+K)-1)\left|\sigma\right|/2\quad . \tag{15}$$

With the set of parameters $\{\alpha=1,\ \beta=0.01,\ \lambda=10\}$, giving $K=-9/11$, σ_1 is a tensile stress less than $|\sigma|/2$, not likely to be responsible for crack nucleation under an applied compressive stress of 0.6 MPa, as observed by Gold (1972).

3.2 Principal stresses in the matrix

Using relations (10), (11), and (12), the stresses in the matrix, expressed in polar co-ordinates related to the local reference frame of the inclusion by (5), become

$$\begin{aligned}&s_{rr}=\sigma\,[-[1+Kg(\xi)]\cos 2\theta\cos 2\varphi+[1-Kg(\xi)]\sin 2\theta\sin 2\varphi\,]/2\,,\\ &s_{\theta\theta}=-s_{rr},\quad s_{zz}=s_{\theta z}=s_{rz}=0\,,\\ &s_{r\theta}=\sigma\,[[1-Kg(\xi)]\sin 2\theta\cos 2\varphi+[1+Kg(\xi)]\cos 2\theta\sin 2\varphi\,]/2\,,\\ &p=\sigma\;[1+2K\cos 2(\theta-\varphi)]\xi^{-2}/2.\end{aligned} \tag{16}$$

The principal stresses at any point (r,θ) are given by

$$\sigma_1=p+\Delta\,,\quad \sigma_2=p-\Delta\,,\ \Delta=\sqrt{s_{rr}^2+s_{r\theta}^2}\quad,\sigma_{zz}=p\,, \tag{17}$$

and the maximum principal stress can be written as

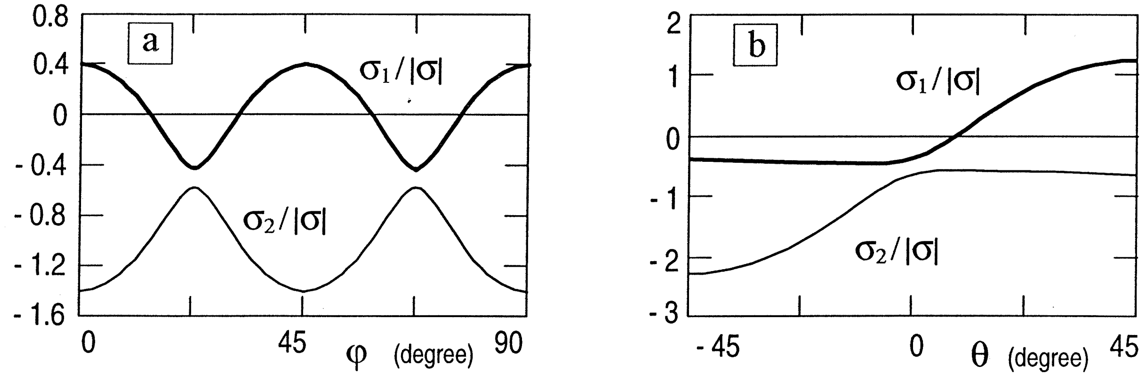

Figure 2. a) principal stresses inside the inclusion as a function of its orientation; b) principal stresses in the matrix along the boundary of a grain oriented at $\varphi = 45°$

$$\sigma_1 = |\sigma| \left[\sqrt{(1-Kg(\xi))^2 + 4Kg(\xi)\cos^2 2\theta} - 2K\cos 2(\theta-\varphi)\xi^{-2} - 1\right]/2 \tag{18}$$

The anisotropy of the grain being such that $\beta \ll 1$ and $\alpha \approx 1$, from relations (3) and (9) it follows, independently of the actual numerical values adopted in section 3.1, that $-1 < K < 0$. Since $g(\xi)$, given by (5), is such that $-1/3 \leq g(\xi) \leq 1$ in the matrix (where $1 \leq \xi \leq \infty$), $1 - Kg(\xi)$ and $1 + Kg(\xi)$ are positive. For a given ξ, the maximum principal stress is maximum when the two first terms in the left side of (18) are maximum. Since K is negative, the first term is maximum for $\cos 2\theta = 0$ if $g(\xi) \geq 0$, or $\cos^2 2\theta = 1$ if $g(\xi) < 0$, and the second for $\cos 2(\theta - \varphi) = 1$. In the first case σ_1, is maximum when $-Kg(\xi) - 2K\xi^{-2}$ has is maximum, equal to $-3K$ for $\xi = 1$, in the second, when $Kg(\xi) - 2K\xi^{-2}$ has is maximum equal to $-4K/3$ ($<-3K$), for $\xi^{-2} = 2/3$.

To summarise, the maximum principal stress in the matrix occurs at the inclusion-matrix interface ($\xi = 1$), for an orientation φ of the inclusion and at an angle θ relative to the basal plane such that $\theta = \varphi = 45 \pm 90°$. The optimum orientation corresponds to the grains well oriented for basal glide. In this configuration the maximum principal stress is

$$\sigma_1 = -3K|\sigma|/2 \tag{19}$$

This stress is tensile, and is obtained at the upper and lower points of the inclusion-matrix interface ($\theta + \varphi = \pm 90°$). Equation (16) with $\cos 2\theta = \cos 2\varphi = 0$ shows that it corresponds to the tangential component $\sigma_{\theta\theta}$, that is the direction of tension is perpendicular to the direction of compression (*i.e.* parallel to the X-axis). For an applied compressive stress of 1MPa, and with the value $K = -9/11$, the tensile stress induced by the anisotropy of the well oriented grains is 1.23 MPa, high enough to nucleate a crack in mode I.

Figure 2b shows the principal stresses σ_1 and σ_2 as a function of the polar angle θ for an easy glide oriented inclusion ($\varphi = 45 \pm 90°$).

4. CRACK NUCLEATION CRITERION

Following Shyam Sunder and Wu (1990), and Wu and Niu (1995), a microcrack is assumed to form in mode I when the local fracture toughness k_I reaches the local critical stress intensity factor k_{IC}. For a precusor of length $2a$, this will happen when $k_1 = \sigma_1(\pi a)^{1/2} = k_{1C}$, that is when the maximum tensile principal stress σ_1 will reaches a critical value defined as $\sigma_C = k_{1C}(\pi a)^{-1/2}$.

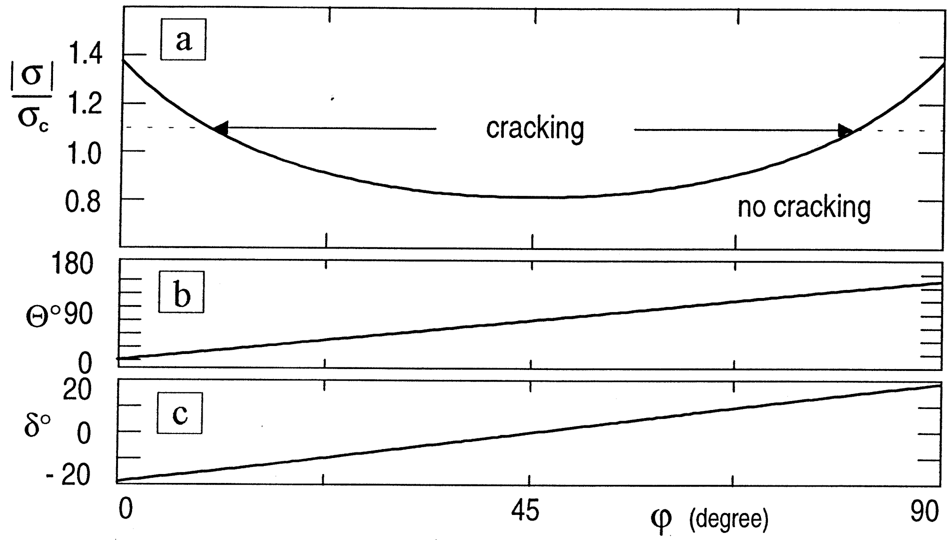

Figure 3. a) criterion for crack nucleation under uniaxial compression $\sigma = -|\sigma|$; b) angular position $\Theta = \theta + \varphi$ (with respect to the X-axis) of the points where σ_1 is maximum; c) angle of the crack with respect to the compression axis (Y-axis), drawn as functions of the crystallographic orientation φ of the grains.

Restricting the analysis to cracks which form at the grain-matrix interface, the proposed criterion derives from equation (18) as

$$|\sigma|[\sqrt{(1-K)^2+4K\cos^2 2\theta}-2K\cos 2(\theta-\varphi)-1]/2 \geq \sigma_c \tag{20}$$

This criterion is satisfied as soon as the maximum of $\sigma_1 = (-3K/2)|\sigma|$ reaches σ_C. The first crack forms in the matrix, parallel to the direction of compression, at position $\theta + \varphi = \pm 90°$, on the boundary of grains oriented at $\varphi = 45 \pm 90°$. With increasing applied stress, the range of grain orientations which satisfy (20) increases. This is illustrated in Figure 3a. This Figure was drawn by adopting the value $K = -9/11$, which corresponds to $\{\alpha = 1,\ \beta = 0.01,\ \lambda = 10\}$, and by calculating the value θ_{max} of θ which maximises the left side of (20), for each value of the orientation φ. Figure 3b shows the angular position $\Theta = \theta_{max} + \varphi$, with respect to the X-axis, of the point of the grain-matrix interface where the maximum principal stress σ_1 is maximum. Figure 3c shows that the cracks, assumed to form in mode I (*i.e.* at 90° from the direction of σ_1), are in a sector at ± 18° from the direction of compression, which is in agreement with observations.

5. DISCUSSION

Creep tests on S2 ice under uniaxial compression perpendicular to the long direction of the grains, show the development of a cracking activity with time for a stress between 0.6 and 1 MPa (Gold, 1972; Plé and Meyssonnier, 1997; Plé, 1998). According to Figure 3a, this minimum stress for crack formation corresponds to a critical stress $\sigma_C = 0.75$ MPa, which is reasonable (adopting Wu and Niu's value $k_{1C} \approx 0.05$ MPa m$^{1/2}$ gives $a \approx 1.4$ mm).

The present analysis is based on the representation of the interaction of a grain with its surroundings by the inclusion model. The underlying assumption is of a statistical nature: a grain with a given orientation stands for the representative of all the grains with the same orientation, while the matrix represents an average of the neighbourhood of these grains. As a consequence, the inclusion model takes into account the local perturbation of the stress field to some extent, but cannot be (and is not) claimed to give an exact representation of the actual intergranular interactions. By nature, it provides a minimum for the stress concentrations induced at the grain scale. The circular shape of the inclusion and the uniform fields of stress and strain-rate inside the inclusion, which result from the solution of the inclusion problem, contribute to minimize further these stress concentrations. In that sense, the present model shows clearly the influence of the viscoplastic anisotropy of ice on the local stress field, without involving processes which could generate stress singularities, such as strain localisation along slip bands or dislocation pile-ups, or geometric singularities at triple junctions.

According to Gold (1972), the shear stress for crack nucleation derived from Stroh's criterion is $\tau_c \approx 0.5$ MPa, and is in agreement with his observations (formation of cracks for stresses greater than 0.6 MPa) if one assumes a uniform stress in the polycrystal (which is equivalent to consider each grain as an isolated crystal). However, from equation (13) the resolved shear stress σ_{xy} acting on the basal plane of an easy glide oriented grain ($\varphi = \pm 45°$) is about $|\sigma|/10$ only (instead of $|\sigma|/2$ corresponding to the far field stress), which is too low to explain crack formation induced by the pile-up mechanism (using Stroh criterion and without taking into account the tensile stress induced by the anisotropic inclusion effect in the matrix).

The present analysis is restricted to a linear viscoplastic behaviour. Taking into account the non-linearity of the ice behaviour would require a heavy numerical treatment in order to obtain results with a statistical meaning. However, the tensile stress at the interface between a 45° oriented grain and the matrix has been obtained with the extension of the anisotropic constitutive law (1) for non-linear behaviour proposed by Meyssonnier and Philip (1999b). The ratio of the viscosity parameters in the planes of isotropy of the grain and of the matrix was given the value 100, according to finite-elements results from Meyssonnier and Philip (1999a) The tensile stress was found to be about 0.6 times that obtained in the linear case, for the same values of the anisotropy parameters ($\alpha = 1$, $\beta = 0.01$). This shows that the stress dependency of the viscosity of the matrix tends to smooth the stress concentrations at the inclusion-matrix interface, as expected, but that the anisotropic inclusion effect should still be considered as an efficient mechanism for crack nucleation.

6. CONCLUSION

An analysis of the stress concentrations arising at the grain scale during steady creep of S2 columnar ice under uniaxial compression has been presented. It is based on the inclusion model in which each grain taken separately is considered to interact with a homogeneous medium which exhibits the mean properties of the polycrystal. Analytical expressions for the stresses have been obtained by assuming each grain as a linearly transversely isotropic medium. The grain anisotropy parameter which characterises the essential feature of the ice viscoplastic anisotropy (*i.e.* the weak resistance to shear parallel to the basal plane) was adjusted by using a self-consistent approach and data from textured ice polycrystal. The analysis shows that the shear stress acting on the basal plane of a grain is maximum for a grain with its *c*-axis at 45° from the direction of compression, but is too low to induce transcrystalline cracking by the dislocation pile-up mechanism. On the other hand, the deformation of such a grain generates a tensile stress in the matrix, which is high enough to nucleate a crack parallel to the direction of loading.

7. REFERENCES

Cole, D.M. 1988. Crack nucleation in polycrystalline ice. Cold Reg. Sci. Technol., 15, 79-87.

Duval, P.; Ashby, M.F.; and Anderman, I. 1983. Rate-controlling processes in the creep of polycrystalline ice. J. Phys. Chem., 87 (21), 4066-4074.

Eshelby, J.D. 1957. The determination of the elastic field of an ellipsoidal inclusion, and related problems. Proc. R. Soc. Lond., A241, 376-396.

Gold, L.W. 1972. The process of failure of columnar-grained ice. Phil. Mag., 26 (2), 311-328.

Gold, L.W. 1997. Statistical characteristics for the type and length of deformation-induced cracks in columnar-grain ice. J. Glaciol., 43 (144), 311-320.

Kalifa, P., Duval, P. and Ricard, M. 1989. Crack nucleation in polycrystalline ice under compressive stress states. Proc. 8th Int. Conf. O.M.A.E.89, The Hague, March 19-23 1989, Vol IV : Arctic and polar technology. ASME, 13-21.

Kamb, W.B. 1961. The glide direction in ice. J. Glaciol., 3 (30), 1097-1106.

Lliboutry, L. 1993. Anisotropic, transversely isotropic non linear viscosity of rock ice and rheological parameters inferred by homogenization. Int. J. Plast., 9, 619-632.

Meyssonnier, J. and Philip, A. 1996. A model for the tangent viscous behaviour of anisotropic polar ice. Ann. Glaciol., 23, 253-261.

Meyssonnier, J. and Philip, A. 1999a .Comparison of Finite-Element and homogenization methods for modelling the viscoplastic behaviour of a S2-columnar ice polycrystal. Ann. Glaciol., 30, In Press.

Meyssonnier, J. and Philip, A. 1999b. Remarks on self-consistent modelling of polycrystalline ice. Proceedings of the Sixth International Symposium on Thermal Engineering and Sciences for Cold Regions, ISTESCR 99. In Press.

Nickolayev, O.Y. and Schulson, E.M. 1995. Grain-boundary sliding and across-column cracking in columnar ice. Phil. Mag. Letters, 72 (2), 93-97.

Picu, R.C. and Gupta, V. 1995. Crack nucleation in columnar ice due to elastic anisotropy and grain boundary sliding. Acta Metall. Mater., 43 (10), 3783-3789.

Pimienta, P., Duval, P. and Lipenkov, V.Y. 1987. Mechanical behavior of anisotropic polar ice. Int. Assoc. Hydrological Sci. (IAHS),170 (Symp. Vancouver 1987 - Physical Basis of Ice Sheet Modelling), 57-66.

Plé, O. 1998. Contribution à l'étude de l'endommagement de la glace de lac en compression uniaxiale. Thèse de l' Université Joseph Fourier, Grenoble I, 233p.

Plé, O. and Meyssonnier, J. 1997. Preparation and preliminary study of structure-controlled S2 columnar ice. J. Phys. Chem.B, 101 (32), 6118-6122.

Shyam Sunder, S. and Wu, M.S.;. 1990. Crack nucleation due to elastic anisotropy in polycrystalline ice. Cold Reg. Sci. Technol., 18, 29-47.

Sinha, N.K. 1984. Intercrystalline cracking, grain-boundary sliding and delayed elasticity at high temperatures.J. Mat. Sci., 19, 359-376.

Stroh, A. N. 1957. A theory of the fracture of metals. Advances in Physics, 6, 418-465.

Wu, M.S. and Niu, J. 1995. Micromechanical prediction of the compressive failure of ice : model development. Mech. Mater., 20 (1), 9-32.

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